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SEMINARIO DE TOPOLOGÍA

USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA
FACULTADE DE MATEMÁTICAS

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Topology Workshop

Hiraku Nozawa (Ritsumeikan Univ.), Kohei Iwamoto (Ritsumeikan Univ.), Álvaro Muñiz Brea (Univ. of Edinburgh), Ramón Barral Lijó (Univ. Politécnica de Madrid), Gabriel Martínez De Cestafe Pumares (Univ. Autónoma de Barcelona), Carlos Meniño Cotón (Univ. de Vigo María José Pereira Sáez (Univ. da Coruña), Isaac Carcacía Campos (USC), Samuel Castelo Mourelle (USC), Alejandro O. Majadas Moure (USC), Ángel Méndez Vázquez (USC).

The purpose of this workshop is to bring together prestigious topologists and students to share a day of presentations and discussion about current trends in Topology and its Applications. Summaries are provided in a separate document.

10:00-10:30 **M.J. Pereira.** *An Application of Algebraic Topology to Economics.*
10:30-11:00 **Á. Méndez**, *Cohomology LS Category.*
11:00-11:30 **I. Carcacía**, *Svarc genus and Cohomological inequalities.*
Coffee break
12:00-12:30 **S. Castelo,** *The Lefschetz number in categories and cellular posets.*
12:30-13:00 **A.O. Majadas**, *An application of topological invariants to sensors and counting.*
13:00-13:30 **C. Meniño**, *Metric invariants from topological data analysis.*
Lunch
15:30-16:00 **K. Iwamoto**, *Birkhoff sections for the geodesic flow on the two-dimensional torus.*
16:00-16:30 **R. Barral**, *Dynamical study of solenoids.*
16:30-17:00 **Á. Muñiz**, *Grupo de cobordismos Lagrangianos.*
Coffee break
17:30-18:00 **G. Martínez De Cestafe**, *Multiplicative aspects of global algebraic K-theory.*
18:00-18:30 **Hiraku Nozawa**, *Information geometry from the coarse viewpoint.*

Data: 22 de Xullo

Lugar: Salón de Graos, Facultade de Matemáticas

Duración: 11 conferencias de ½ hora

Hora: 10:00–13:30, 15:30-18:30

UNIVERSIDADE DA CORUÑA
USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA
Universidade Vigo
CITMAGA CENTRO DE INVESTIGACIÓN E TECNOLOXÍA MATEMÁTICA DE GALICIA

Topology Workshop – Summaries

Ramón Barral Lijó (Univ. Politécnica de Madrid). *Dynamical study of solenoids.*
We will review the classical theory of solenoids before presenting the ongoing task of studying their induced dynamical systems and their applications to the classification of solenoids.

Isaac Carcacía Campos (USC). *Svarc genus and Cohomological inequalities.*
The notion of Svarc genus was introduced by Svarc in the 60s to measure how far a fiber map is from having a section. The aim of this talk is to explain this concept, its evolution and show how it is related to important notions in algebraic topology such as fibration and homotopy. Next we will develop some applications to the study of other homotopical invariants that measure how many open sets one must have in order to have a cover satisfiying some property, such as the LS-category and topological complexity. Moreover, we will see how those invariants are related to algebraic properties that appear using cohomology.

Samuel Castelo Mourelle (USC). *The Lefschetz number in categories and cellular posets.*
We will introduce the notion of the Euler Characteristic of a small finite category in an array of different forms. Some of them don't always exist for certain categories. However, when one is able to define them in a particular category they coincide.

Then, we want to develop the Lefschetz number of a certain type of posets, called cellular posets. This posets have a unique way of computing their homology, directly counting elements on the poset, instead of having to go through the order simplicial complex which make it simpler.

Kohei Iwamoto (Ritsumeikan University). *Birkhoff sections for the geodesic flow on the two-dimensional torus.*
The Birkhoff sections are useful for investigating dynamical systems on three-dimensional manifolds. It was introduced to study the topological properties of solutions to the equations of motion. In this talk, we explain how to determine the first return map of some Birkhoff sections for the geodesic flow of two-dimensional torus.

Alejandro O. Majadas Moure (USC). *An application of topological invariants to sensors and counting.*
Certain topological invariants, such as the combinatorial Euler characteristic can be used to obtain applications on counting systems. Nevertheless, if se consider only the Euler characteristic, some restrictions appear relative to the shape of the object. So, the natural idea to generalize this study and solve these restrictions is to consider the Lefschetz number of a homeomorphism instead of only the Euler characteristic. Is there where appears the Lefschetz combinatorial number.

Gabriel Martínez De Cestafe Pumares (Univ. Autónoma de Barcelona). *Multiplicative aspects of global algebraic K-theory.*
Global algebraic K-theory is a refinement of algebraic K-theory due to Stefan Schwede, where the adjective "global" refers to simultaneous and compatible actions of all finite groups. His construction takes as input a so-called parsummable category C and produces a symmetric spectrum $K_{gl}(C)$. One can then look at the collection of 0th equivariant homotopy groups of $K_{gl}(C)$ to recover precise information about C. Global algebraic K-theory admits an extra layer of structure: if a parsummable category C comes equipped with certain multiplicative structure, then $K_{gl}(C)$ can be upgraded into a ring spectrum. In this case, the 0^{th} equivariant homotopy groups of $K_{gl}(C)$ become rings and their multiplication carries information about the multiplicative structure of C. I will explain these ideas
in the talk, taking as example the global algebraic K-theory of a commutative ring.

Carlos Meniño Cotón (Univ. de Vigo). *Metric invariants from topological data analysis.*
Under some conditions, the Vietoris Rips complex constructed on sufficiently dense set of points on a closed Riemannian manifold is homotopic to to the manifold itself. This is exploited from topological data analysis to obtain information of unknown manifolds from empirical samples. In this work we deal with the inverse problem: what kind of extra data can be found of a known Riemannian manifold from persistence diagrams over sufficiently dense cloud points.

It is clear that the persistence diagram of a uniform sampling over a Riemannian manifold is an statistical object. It is known that the mean of this random object exists. It is clear that this will be a metric invariant of the

Riemannian manifold. Although persistent classes only deal with homology, the birth and death times is strongly related with both the local and global behaviour of the metric, so it is expected that it captures some metric information which is not easy to extract from other invariants like curvature.

As a proof of concept test, we have made estimations of these invariants via bootstrap technique in 3-dimensional lens spaces and hyperbolic 3-manifolds with promising results. These are families of manifolds where we can find pairs of manifolds that are not easy to distinguish. The case of closed hyperbolic 3-manifolds is very interesting since rigidity theorems imply that metric invariants are, in fact, topological invariants.

This is a joint work with my TFG student Pedro Tarrío Mallou.

Álvaro Muñiz Brea (Univ. of Edinburgh). *Grupo de cobordismos Lagrangianos.*
Un cobordismo Lagrangiano es la versión simpléctica de un cobordismo en topología geométrica/algebraica. Introducidos por Arnold en los años 80 como un intento de clasificación de subvariedades Lagrangianas, más recientemente el trabajo de Biran-Cornea en 2013 muestra que estos objetos geométricos detectan la estructura triangulada de la categoría de Fukaya (invariante simpléctico más potente hasta la fecha). Tras repasar conceptos básicos en topología simpléctica, en esta charla introduciremos los cobordismos Lagrangianos y estudiaremos el caso de un 2-toro.

Hiraku Nozawa (Ritsumeikan University). *Information geometry from the coarse viewpoint.*
A statistic on a statistical model is sufficient if it has no information loss, namely, the Fisher metric of the induced model coincides with that of the original model due to Kullback and Ay-Jost-Lê-Schwachhöfer. We introduce a quantitatively weak version of sufficient statistics such that the Fisher metric of the induced model is bi-Lipschitz equivalent to that of the original model. We characterize such statistics in terms of the conditional probability or by the existence of a certain decomposition of the density function in a way similar to characterizations of sufficient statistics due to Fisher-Neyman and Ay-Jost-Lê-Schwachhöfer.
This is a joint work with Kaori Yamaguchi.

María José Pereira Sáez (Univ. da Coruña). *An Application of Algebraic Topology to Economics.*
Farber's Topological Complexity is a topological invariant that measures how hard is to plan continuous motions in terms of the configuration space of the system. In this talk, we will apply ideas developed in the field of Topological Complexity to Social Choice Theory. This branch of Economics studies how to aggregate the preferences of various individuals to obtain a common group preference. We will present here the notion of Social Choice Complexity and its higher version relating it with the Higher Topological Complexity and its symmetric version.

Ángel Méndez Vázquez (USC). *Cohomology LS Category.*
The Lusternik-Schnirelmann category of a topological space is a well-known homotopy invariant, yet difficult to calculate. It originally appeared in variational calculus as a lower bound for the number of critical points of a function defined on a manifold. Recently, it has gained popularity due to its connection with some problems related to robotics. As a result, different versions and generalizations have emerged in finite, combinatorial, geometric, simplicial, and small category contexts. The aim of the talk is to present a lower bound: the cohomology category. We will also discuss the simplicial category and its corresponding cohomology version, which can be calculated using computational methods for any triangulable space.