Book of Abstracts

Symmetry and shape

23 - 27 September 2024

Santiago de Compostela, Spain

http://xtsunxet.usc.es/symmetry2024/

Welcome

According to Felix Klein, geometry is the study of those properties in space that are invariant under a given transformation group. Intuitively, symmetry is the correspondence of shape at every point of a space. An interesting problem in geometry and many physical sciences is to determine the symmetries of a space from its shape.

The aim of this conference is to gather experts in the study of symmetry in Differential Geometry, whilst we celebrate Eduardo Garca Ro's 60th birthday. The conference will revolve around the study of curvature, homogeneous and symmetric spaces, Riemannian submanifold geometry, and other related topics in Differential Geometry and Geometric Analysis.

Contents

Organization

Organizing committee

Miguel Brozos Vzquez, Universidade da Corua, Spain
Jos Carlos Daz Ramos, Universidade de Santiago de Compostela, Spain
Miguel Domnguez Vzquez, Universidade de Santiago de Compostela, Spain
Ixchel Dzohara Gutirrez Rodrguez, Universidade de Vigo, Spain
Vctor Sanmartn Lpez, Universidade de Santiago de Compostela, Spain
M. Elena Vzquez Abal, Universidade de Santiago de Compostela, Spain
Ramn Vzquez Lorenzo, Consellera de Educacin, Xunta de Galicia, Spain

Sponsors

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Project ED431C 2023/31, Xunta de Galicia, Spain
Centro de Investigacin e Tecnoloxa Matemtica de Galicia (CITMAga), Spain
Red Espaola de Anlisis Geomtrico

Contact information

congreso.symmetry2024@usc.gal

Schedule

The scientific programme is composed of lectures by the main speakers and talks. All lectures will take place in Aula Magna of the Faculty of Mathematics.

The conference dinner will take place at *Hotel Palacio del Carmen* on Thursday 26th at 20:00.

Practical information

Internet connection

You can connect via **eduroam**, if your affiliation supports it and you have your devices correctly configured.

Restaurants

There are several options to have lunch near the conference venue:

- Some restaurants near the campus, such as *Sicilia in Bocca*, *Santos*, *Xugo*, *Xantar* or *Altamira*, offer lunch menus for around 12-15 euros.
- Downtown and in the historic center there are many kinds of restaurants. Vegan and vegetarian options can be found, for example, at *Boca a boca*, *Malak Bistro*, *A Tulla* or *The Veggie Carmen*. Many restaurants offer typical Galician food, such as *O Dezaseis* or *María Castaña*.
- The cafeteria-canteens of the Faculty of Mathematics (same building as the conference), of the Escola Técnica Superior de Enxeñaría - ETSE (200m), or the cafeterias Fonseca and Rodríguez-Cadarso (300m) offer menus for 8.50 euro. Because of students' schedule, it is recommended to have lunch before 13:30.

Abstracts

Main speakers

Holomorphic pseudosymmetric Kaehler manifolds

Alma Albujer
(Universidad de Alicante, Spain)

Natural symmetries of Riemannian manifolds, and in particular of Kaehler manifolds, are determined by the complexity of its Riemann curvature tensor. Specifically, we can consider constant holomorphic sectional curvature, locally symmetric, semisymmetric and holomorphically pseudosymmetric Kaehler manifolds. This last family is defined in a slightly different way to the classical notion of pseudosymmetric Riemannian manifolds proposed by Deszcz. In the first part of the talk we will give some geometric interpretations of certain notions related to these concepts of symmetry. We will also study some relations between the notions of holomorphic pseudosymmetry and pseudosymmetry in the sense of Deszcz via the so called double sectional curvatures of Deszcz. Finally, we will study semisymmetric and holomorphically pseudosymmetric complex hypersurfaces in a complex space form. The results presented in this talk are part of a joint work with Jorge Alcázar and Magdalena Caballero.

Geometric analysis of the Lorentzian distance function

Luis Alías
(Universidad de Murcia, Spain)

Let $\overline{M}$ be a spacetime, and consider either the Lorentzian distance function from a point $p \in \overline{M}$, or the Lorentzian distance function from an achronal spacelike hypersurface $\Sigma \subset \overline{M}$. Under suitable conditions these Lorentzian distance functions are differentiable at least in a "sufficiently near chronological future" of the point p or of the hypersurface Σ, so that some classical analysis can be done on those functions. In 2003, Erkekoglu, García-Río and Kupeli [1] established the basis for the comparison analysis of the (Lorentzian) Hessian and Laplacian operators of these functions. In this talk we will study the Lorentzian distance function restricted on certain spacelike hypersurfaces and spacelike submanifolds of $\overline{M}$ and derive sharp estimates for the mean curvature of such submanifolds under appropriate hypotheses on the curvature of the ambient spacetime. Our results are obtained as an application of the aforementioned comparison results for the Lorentzian distance as well as the generalized Omori-Yau maximum principle for the (Riemannian) Hessian and Laplacian operators.

References:

[1] F. Erkekoglu, E. García-Río, D. N. Kupeli: *On level sets of Lorentzian distance function*, General Relativity and Gravitation **35** (2003), 1597-1615.

Shape enhancement on manifolds: regularity issues for the ROF model

Esther Cabezas-Rivas
(Universitat de València, Spain)

The Rudin-Osher-Fatemi functional (ROF) is the most popular variational model for shape denoising. It is based on the total variation functional plus a fidelity term that penalizes lack of proximity to a given image. The latter is encoded through a function f defined on a compact surface with a boundary, which takes values on an open Riemannian n-manifold.

The corresponding Euler-Lagrange equations are of independent interest, since they can be interpreted as an eigenvalue problem for the Riemannian analogue of the 1-Laplacian. From this point of view, these are elliptic systems that have certain similarities with the theory of harmonic (or p-harmonic) maps between manifolds.

We prove the existence and uniqueness of minimizers of such a functional, under suitable curvature conditions. Then we present a series of regularity results results on the system of PDEs associated to a relaxed ROF-like functional with a Neumann condition. The two key ingredients are estimates for the gradient that are deduced from the application of a Caccioppoli-type inequality obtained by the authors themselves [1], as well as the adaptation of classical (Euclidean) regularity results for for Neumann-type problems.

Finally, after bootstrapping and a limiting process, we apply these results to obtain Lipschitz regularity of the minimizers the manifold-valued ROF model.

This is a joint work with Salvador Moll and Vicent Pallardó-Julià.

References:

[1] E. Cabezas-Rivas, S. Moll, V. Pallardó-Julià: Partial regularity for manifold constrained quasilinear elliptic systems, *Nonlinear Anal.* **249** (2024), paper no. 113643, 5 pp.

On the Geometry of Homogeneous Lorentzian Three-Manifolds

Giovanni Calvaruso
(Università del Salento, Italy)

Three-dimensional homogeneous Riemannian and Lorentzian manifolds have been extensively studied for many years by several authors. Although they have been investigated under many different points of view, they are still an active research topic. In the first part of this talk I shall give an overview of the main results concerning the geometry of homogeneous Lorentzian three-manifolds. For each property, these results will be compared with their Riemannian counterpart, that is, with the corresponding feature for homogeneous Riemannian three-manifolds. We shall start from the classification of Riemannian and Lorentzian manifolds and then proceed to illustrate several geometric properties, like the classification of Einstein-like metrics, homogeneous geodesics and Ricci solitons. In the second part I shall focus on some recent results about homogeneous Lorentzian three-manifolds, with particular regard to Lorentzian Bianchi-Cartan-Vranceanu spaces and the classification of homogeneous Lorentzian structures.

Brackets

Manuel de León
(ICMAT, Spain)

The notion of Poisson bracket is the translation of the symplectic structure to the algebra of functions of the manifold, and plays a crucial role in the study of the integrability of a Hamiltonian system by means of the famous Liouville-Arnold theorem. Throughout this lecture, we will describe that role and how to extend it to contact Hamiltonian systems and multisymplectic field theories.

Groups of symmetries and the dynamics of the geodesic flow

Ana Ferreira
(Universidade do Minho, Portugal)

We will discuss the geometry of the geodesic flow of Lie groups with left-invariant pseudo-Riemannian or holomorphic-Riemannian metrics. As well as recalling some beautiful and seminal classical theory, we will present some recent developments in this topic. This talk will be based on a series of joint works with S. Chaib, A. Elshafei, H.Reis, M. Sanchez and A. Zeghib.

Polygons and G_2-symmetry

Luis Hernández Lamoneda
(CIMAT, Mexico)

Think of a sphere of radius r rolling along a closed geodesic polygon, drawn on a second sphere of radius R. Assume the rolling is *without slipping nor twisting*. We look at those polygons such that the final and initial positions of the small sphere (after traversing the polygon) are identical. These trajectories correspond to certain *horizontal* closed curves on the 5-dimensional configuration space Q, that parametrizes all possible contact positions between the spheres. When $R/r = 3$, the group of symmetries of this geometric scenario is G_2 (the non-compact, split version of this group).

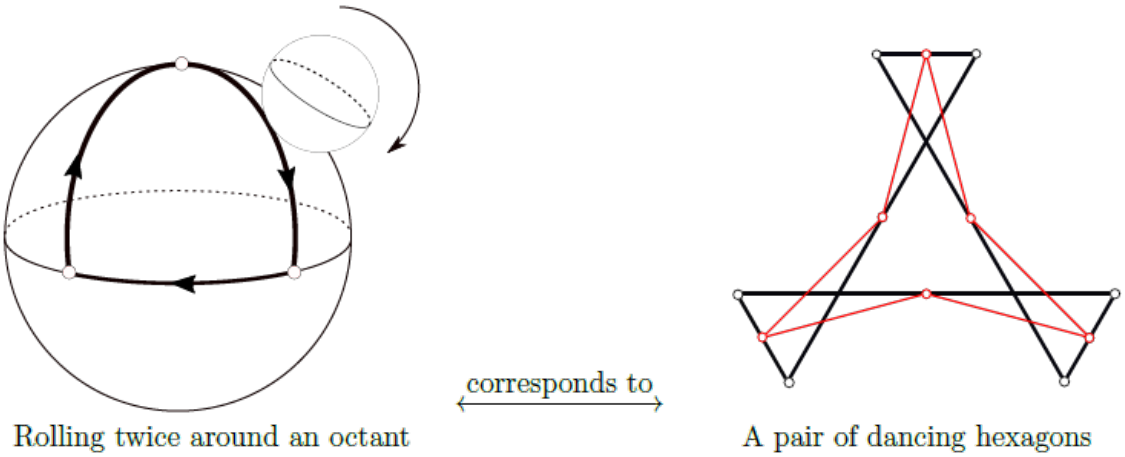
I'll give a correspondence between such geodesic spherical polygons and pairs of polygons in the projective plane $\mathbb{RP}^2$, satisfying a certain projective geometric condition. This hints to a possible characterization of G_2 using solely projective geometric quantities.

This is joint work with Gil Bor, also of CIMAT, Guanajuato, Mexico.

Lorentzian homogeneous structures with indecomposable holonomy

Thomas Leistner
(University of Adelaide, Australia)

The results in this talk are motivated by the question to which extent the curvature of a homogeneous space G/H is determined by algebraic conditions on the isotropy representation of H. While there are many Riemannian homogeneous spaces with irreducible isotropy, the Lorentzian case is very rigid: Zeghib proved that a Lorentzian homogeneous



space with irreducible isotropy has constant sectional curvature. This indicates that for indefinite metrics the weaker notion of indecomposability is more relevant. It means that the isotropy representation of H may have invariant subspaces but only those on which the metric is degenerate. Due to a lack of examples, and encouraged by Lorentzian symmetric spaces, we conjecture that a reductive Lorentzian homogeneous space with indecomposable isotropy is a plane wave. In the talk we will present an important step towards this conjecture, namely that a Lorentzian manifold that admits a homogeneous structure (aka an Ambrose-Singer connection) with indecomposable holonomy is a plane wave. The existence of an Ambrose-Singer connection is almost equivalent to reductive homogeneity and its holonomy closely related to the isotropy of the homogeneous space. En route to this result we prove several results about locally symmetric connections and connections with parallel torsion.

This is joint work with Steven Greenwood.

Reduction of time-dependent Hamiltonian systems in the presence of a Hamiltonian symmetry Lie group

Juan Carlos Marrero
(Universidad de la Laguna, Spain)

In this talk, I will present a reduction process for a time-dependent Hamiltonian system in the presence of a Hamiltonian symmetry Lie group. This means that we have a Lie group action such that the geometry and the dynamics associated with the time-dependent system are invariant. In our approach, the dynamics is a vector field which generates the kernel of a presymplectic structure of corank 1. After reduction and under appropriate regularity conditions, we will see that the reduced dynamics also generates the kernel of a reduced presymplectic structure of corank 1. Our method solves some problems in a previous (cosymplectic) reduction process which was proposed by Albert [1]. In addition, it may be applied in the reduction of some time-dependent Hamiltonian systems for which Albert reduction does not work. The results presented in this talk are part of a work in progress in collaboration with I Gutiérrez, D Iglesias and E Padrón.

References:

[1] C. Albert: Le théoréme de réduction de Marsden-Weinstein en géométrie cosymplectique et de contact, *J. Geom. Physics*, **6** (4), (1989), 627-649.

On the conformal geometry of quasi-umbilical timelike surfaces

Lorenzo Nicolodi
(Università di Parma, Italy)

We study the conformal Lorentz geometry of quasi-umbilical surfaces in the Einstein universe, the conformal compactification of Minkowski 3-space. By a quasi-umbilical surface it is meant a timelike immersion of a surface X in the Einstein universe whose shape operator at any point of X is non-diagonalizable over $\mathbb{C}$. We show that quasi-umbilical surfaces are isothermic, that their conformal Gauss map is harmonic, and that their conformal deformations depend on one arbitrary function in one variable. We then prove that quasi-umbilical surfaces arise from suitable null curves in the neutral space form $S^{2,2} = \{x \in \mathbb{R}^{2,3} \mid \langle x, x\rangle = 1\}$. This is joint work with E. Musso and M. Pember.

Marsden theorem and the completeness of left-invariant semi-Riemannian metrics on Lie groups

Miguel Sánchez
(Universidad de Granada, Spain)

The study of invariant Riemannian metrics in a Lie group G goes back to Euler's study of rigid solid motion. When considering indefinite semi-Riemannian metrics, a noticeable possibility is geodesic incompleteness. Our main aim is to give a sufficient condition for the geodesic completeness of all the left-invariant indefinite metrics on G, which is fulfilled and generalizes all the known cases so far. This condition, which might have interest on its own right, means the at most affine growth of the norm of the adjoint representation with respect to any auxiliary left invariant positive definite metric. We will follow a heuristic approach starting at a celebrated theorem by Marsden (compact homogeneous semi-Riemannian manifolds are geodesically complete) and providing a variant of his proof.

The talk is based on recent research with A. Elshafei, A.C. Ferreira and A. Zeghib (arXiv:2308.16513), already published electronically at Trans. Amer. Math. Soc.

Symmetry and supersymmetry of the angular momentum (reloaded)

Adolfo Sánchez-Valenzuela
(CIMAT, Mexico)

The geometry and the topology of the Hurwitz-Hopf map are reviewed in a rather elementary way in order to produce harmonic wave functions that describe integer and half-integer valued states of the angular momentum in a unified fashion. The classical spherical harmonics are recovered as the space of integer-valued states of the angular momentum. The results are applied to the Schrödinger Equation for the hydrogen atom. It is shown how the Laplacian depends on the Euler angle that classically defines a top's spin, and that the atom's quantum energy depends on both, the integer and the half-integer values of the angular momentum.

This presentation is based on a joint work with Sergio Hojman (Universidad Adolfo Ibáñez and Universidad de Chile) and Eduardo Nahmad (Instituto de Ciencias Nucleares de la UNAM) [https://doi.org/10.1088/1402-4896/ace08e]

Hermitian geometry of complex quotient manifolds with trivial canonical bundle

Luis Ugarte
(Universidad de Zaragoza, Spain)

Complex manifolds with trivial canonical bundle, endowed with a (possibly non-Kähler) special Hermitian metric, play an important role both in geometry and in physics. A source of interesting examples is provided by the quotients of Lie groups G by co-compact lattices, where the Lie algebra of G has a complex structure with non-zero closed $(n,0)$-form. In this talk we will focus on the classification of such unimodular Lie algebras in six dimensions, together with the existence of special Hermitian metrics on the corresponding compact complex quotient manifolds. In the balanced setting, these spaces provide many invariant solutions of the Hull-Strominger system, even with non-flat connection ∇ on the tangent bundle being Hermitian-Yang-Mills. We will show that such solutions only exist on three specific spaces, which consist of a nilmanifold, a solvmanifold and a quotient of SO(3,1), and moreover ∇ is given by the Bismut connection. This talk is based on joint works with A. Otal and R. Villacampa.

Talks

Torsion-free connections with prescribed curvature

Efraín Basurto-Arzate
(Technische Universität Dortmund, Germany)

In classical Differential Geometry a central object of study is that of a connection. On the tangent bundle of a smooth manifold a connection defines two particular tensor fields: the torsion and the curvature. A connection with vanishing torsion tensor is said to be torsion-free. One could now ask, whether there is a way to know to what extent the curvature tensor prescribes the connection. Concretely, we consider the problem: given an algebraic curvature map -a map which at each point satisfies the algebraic Bianchi identity-, when can one guarantee, that it actually arises from the curvature tensor of a torsion-free connection? In this talk we provide necessary and sufficient conditions that locally settle this question. Thereafter we will discuss some consequences of these results in the direction of holonomy theory.

Quadratic curvature functionals and four-dimensional homogeneous metrics

Sandro Caeiro-Oliveira
(Universidade de Vigo, Spain)

Einstein metrics are the critical metrics for the Hilbert-Einstein functional when restricted to constant volume variations. This functional is defined in terms of the scalar curvature, which is the only scalar curvature invariant of order one. If we consider second-order curvature invariants, the resulting functionals are the quadratic curvature functionals.

In dimensions 3 and 4, Einstein metrics are critical for all quadratic curvature functionals. In this talk, we will give all the homogeneous metrics that are critical for some quadratic curvature functional in dimension 4. Moreover, we will show the relation between Ricci solitons and critical metrics, while we point out some differences with the 3-dimensional setting.

This presentation is based on a joint work with Miguel Brozos Vázquez, Eduardo García Río and Ramón Vázquez Lorenzo [doi.org/10.1090/tran/9219].

Ricci solitons as submanifolds of complex hyperbolic spaces

Ángel Cidre Díaz
(Universidade de Santiago de Compostela, Spain)

As a consequence of the recently solved Alekseevsky conjecture [1], and a result lying at the intersection of Ado's theorem for Lie algebras and the Nash embedding theorem [2], any expanding homogeneous Ricci soliton can be found, up to isometry, as a Lie subgroup of the solvable Iwasawa group associated with a symmetric space of non-compact type, considered with the induced metric. Motivated by this fact, we have addressed the classification of homogeneous Ricci solitons arising as Lie subgroups of the

solvable Iwasawa groups of complex hyperbolic spaces. We also analyse the minimality of the examples obtained.

References:

[1] C. Böhm, R. Lafuente: Non-compact Einstein manifolds with symmetry, *J. Amer. Math. Soc.* **36** (3), 591-651.

[2] M. Jablonski: Einstein solvmanifolds as submanifolds of symmetric spaces, arXiv:1810.11077.

Symmetry in CAT(0) spaces of curvature bounded from below

Maciej Czarnecki
(Uniwersytet ódzki, Poland)

We consider metric analog of a symmetry (reflection) through a totally geodesic subspace of a metric space X which is CAT(0) but its triangles are sufficiently thick (negative metric curvature is bounded from below). We prove that such a symmetry is a quasi-isometry of X and study the reflection case to obtain results analogous to properties of isometries in model spaces. Furthermore, we describe quasi-reflections in quasi-geodesic subspaces.

Integrable systems, Painlevé VI and explicit solutions to the anti-self dual Einstein equation via radicals

Owen Dearricott
(La Trobe University, Australia)

Though Einstein's equation is well studied, relatively few Einstein metrics have been written in terms of explicit formulae via radicals. In this talk we discuss many such examples that occur as anti-self dual Einstein metrics and describe their singularities.

The construction heavily relies upon the theory of isomonodromic deformation and related algebraic geometry developed by N.J. Hitchin in the 1990s and the equivalence of the anti-self dual Einstein equation to a certain Painlevé VI equation under some symmetry assumptions discovered by K.P. Tod. The solution to Painlevé VI is achieved through a relation of its solution to pairs of conics obeying the Poncelet's porism by exploiting Cayley's criterion.

In this talk we discuss some important cases that are not well fleshed out in the literature, such as the solution of Painlevé VI associated with the Poncelet porism where the inscribing-circumscribing polygons have an even number of sides.

Moreover, we provide some explicit metrics with unusual cone angle singularities along a singular real projective plane that were speculated about by Atiyah and LeBrun and discuss their sectional curvature.

Totally geodesic submanifolds of the symmetric space $G_2/SO(4)$

Cristina Draper Fontanals
(Universidad de Málaga, Spain)

We provide an independent proof of the classification of the maximal totally geodesic submanifolds of the symmetric space $G_2/SO(4)$, jointly with very natural descriptions of all of these submanifolds, in terms of associative subalgebras and in terms of Grassmannians.

Volume of vector fields on a Riemannian manifold: some open problems

Olga Gil Medrano
(U. de Valencia, Spain)

Vector fields are considered as embeddings of the manifold into its tangent bundle $V\colon M \to TM$. For a Riemannian metric g on M, the volume of V is defined as the volume of the corresponding submanifold $V(M) \subset (TM, g^S)$, where g^S is the Sasaki metric which is defined by g and its covariant derivative. The aim of the talk is to exhibit some of the many open problems on this subject contained in our recent book [1], in the hope of encouraging further research.

References:

[1] O. Gil-Medrano: *The Volume of Vector Fields on Riemannian Manifolds. Main Results and Open Problems*, Lecture Notes in Mathematics 2336, Springer, Berlin, 2023.

Conformally Einstein four-dimensional Lie groups

Ixchel Dzohara Gutiérrez Rodríguez
(Universidade de Vigo, Spain)

Einstein manifolds, which are critical points of the Hilbert-Einstein functional, play a central role in pseudo-Riemannian geometry. One strategy to construct Einstein metrics involves deforming a given metric by a conformal factor so that the resulting metric is Einstein. In this talk, we will follow this approach with a special emphasis on dimension four, providing a complete description of four-dimensional conformally Einstein Lorentzian Lie groups.

This is a joint work with E. Calviño-Louzao, E. García-Río and R. Vázquez-Lorenzo.

Nearly Kähler geometry and totally geodesic submanifolds

Juan Manuel Lorenzo Naveiro
(Universidade de Santiago de Compostela, Spain)

An almost Hermitian manifold (M^{2n}, J) is said to be nearly Kähler if the covariant derivative of J is totally skew-symmetric. It can be shown that if $M \neq \mathsf{S}^6$ is a simply connected six-dimensional strict nearly Kähler manifold, then one can rescale the metric on M so that its Riemannian cone $\widehat{M}$ has special holonomy G_2. A theorem of Butruille asserts that the simply connected homogeneous strict nearly Kähler manifolds of dimension six are the round sphere S^6, the space $\mathsf{F}(\mathbb{C}^3)$ of full flags in $\mathbb{C}^3$, the complex projective space $\mathbb{C}\mathsf{P}^3$ and the almost product $\mathsf{S}^3 \times \mathsf{S}^3$. These spaces belong to the general class of naturally reductive homogeneous spaces, whose geometry can be understood in purely Lie-algebraic terms.

The purpose of this talk is to discuss the classification of totally geodesic submanifolds of the (non-symmetric) homogeneous strict nearly Kähler 6-manifolds, as well as their G_2 cones. To this end, we will develop the algebraic framework needed to attack the problem, and later on we will exhibit the examples that appear in each case.

This is a joint work with Alberto Rodríguez-Vázquez (KU Leuven).

Conformal structures with an infinitesimal symmetry

Omid Makhmali
(UiT - The Arctic University of Norway, Norway)

We interpret the property of having an infinitesimal symmetry as a variational property in certain geometric structures. This is achieved by establishing a one-to-one correspondence between a class of cone structures with an infinitesimal symmetry and geometric structures arising from certain systems of ODEs that are variational. Such cone structures include conformal pseudo-Riemannian structures and distributions of growth vectors (2,3,5) and (3,6). The correspondence is obtained via symmetry reduction and quasi-contactification. Subsequently, we give an exhaustive construction of cone structures with more specific properties, such as having a null infinitesimal symmetry, being foliated by null submanifolds, or having reduced holonomy to the appropriate contact parabolic subgroup. As an application, we show that chains in integrable CR structures of hypersurface type are Finsler metrizable. This is a joint work with Katja Sagerschnig.

Cohomology of Quaternionic Foliations and Orbifolds

Rouzbeh Mohseni
(Institute of Mathematics, Polish Academy of Sciences, Poland)

First I will briefly discuss quaternionic geometry and quaternion Kähler manifolds and then define a transversely quaternion Kähler foliation. Then I will present our result regarding foliated versions of the now classical results of V.Y. Kraines and A Fujiki on the cohomology of quaternion Kähler manifolds. Finally, as any orbifold can be realized as the leaf space of a suitably defined Riemannian foliation I will discuss our results for quaternion orbifolds.

This talk is based on joint work with R. A. Wolak [1].

References:

[1] R. Mohseni, R. A. Wolak: Cohomology of quaternionic foliations and orbifolds, arXiv:2202.02733.

The vacuum weighted Einstein field equations: Properties and rigidity of solutions

Diego Mojón Álvarez
(Universidade de Santiago de Compostela, Spain)

A standard Lorentzian spacetime (M, g) is generalized by introducing a positive density function h, giving rise to a smooth metric measure spacetime $(M, g, h\, dvol_g)$. The geometric features of such spacetimes may be studied through objects called weighted invariants, which generalize the usual curvature-related tensors of a semi-Riemannian manifold while including information on the density function.

In this talk, we present a suitable generalization of the Einstein tensor to smooth metric measure spacetimes (the weighted Einstein tensor) and a variational approach that yields its associated field equations.

We study both the isotropic (∇h lightlike) and non-isotropic (∇h timelike or spacelike) solutions to the resulting vacuum equations and obtain local classification results for locally conformally flat solutions and for 4-dimensional solutions with harmonic curvature tensor. The first condition results in a very rigid geometry, while the second one presents more flexibility. In the case of harmonic curvature, the geometry of solutions depends on the Jordan form of the Ricci operator. All solutions present real Ricci eigenvalues, and they are realized either on specific non-isotropic warped product structures with diagonalizable Ricci operator, or on different families of Kundt spacetimes, including pp-waves and plane waves.

References:

[1] M. Brozos-Vázquez, D. Mojón-Álvarez: The vacuum weighted Einstein field equations on pr-waves, arXiv:2407.10535 [math.DG] (2024).

[2] M. Brozos-Vázquez, D. Mojón-Álvarez: Vacuum Einstein field equations in smooth metric measure spaces: the isotropic case, *Class. Quantum Grav.* **39** (13) (2022) 135013, 20 pp https://doi.org/10.1088/1361-6382/ac72e9.

Vector fields and magnetic maps

Marian Ioan Munteanu
(University "Al.I.Cuza" of Iasi, Romania)

This talk is based on some joint papers with J. Inoguchi, Department of Mathematics, Hokkaido University, Japan.

In our paper [1] we define the notion of magnetic map as a generalization of both magnetic curves and harmonic maps. A magnetic map is obtained as critical points of the LH functional, that is the energy functional together with a potential part.

As a vector field can be thought of as a map from the manifold to its tangent bundle and since the tangent bundle carries a natural magnetic field obtained from its almost Kaehlerian structure, we may ask when a vector field is a magnetic map?

Furthermore, we show that a unit vector field on an oriented Riemannian manifold is a critical point of the Landau Hall functional if and only if it is a critical point of the Dirichlet energy functional. Therefore, we provide a characterization for a unit vector field to be a magnetic map into its unit tangent sphere bundle.

Then, we classify all magnetic left invariant unit vector fields on 3-dimensional Lie groups.

References:

[1] J. Inoguchi, M.I. Munteanu: Magnetic maps, *Internat. J. Geom. Methods Mod. Phys.* **11** (2014) 6, art. n.1450058.

[2] J. Inoguchi, M.I. Munteanu: New examples of magnetic maps involving tangent bundles, *Rend. Semin.Mat. Univ. Politec. Torino* **73/1** (2015) 3-4, 101-116.

[3] J. Inoguchi, M.I. Munteanu: Magnetic vector fields: New examples, *Publ. Inst. Math. Beograd* **103** (117) (2018), 91-102.

[4] J. Inoguchi, M.I. Munteanu: Magnetic unit vector fields, *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas* **117** (2023) 2, art. 71.

Fat equator effect and minimality in immersions and submersions of the Sphere

Vicente Palmer
(Universitat Jaume I, Spain)

Inspired by the equatorial concentration of measure phenomenon in the Sphere $S^n(1)$, a result which is deduced from the general, (and intrinsic), concentration of measure in $S^n(1)$, we shall describe in this talk an "extrinsic" equatorial concentration of measure satisfied by the closed, (compact without boundary), isometric and minimal immersions of the Sphere. If time permits, we shall talk too about the same result stated for compact and minimal Riemannian submersions of the Sphere.

Totally Geodesic and Parallel Hypersurfaces of Cahen-Wallach Spacetimes

Lorenzo Pellegrino
(Università del Salento, Italy)

A submanifold M of a pseudo-Riemannian manifold is *totally geodesic* if its second fundamental form h vanishes and, more in general, a submanifold is said to be *parallel* if h is covariantly constant. The parallelism of the second fundamental form may be seen as the extrinsic analogue of local symmetry. Indeed, parallel hypersurfaces of a locally symmetric ambient space are again locally symmetric. In recent years, totally geodesic and parallel hypersurfaces of large classes of spacetimes have been studied, for example in the four dimensional oscillator groups, Gödel-type and Siklos spacetimes.

We focus on indecomposable symmetric Lorentzian manifolds of non-constant sectional curvature, known as *Cahen-Wallach spacetimes*. We shall provide the complete classification and explicit description of totally geodesic hypersurfaces of such spaces. Moreover, we investigate parallel ones, obtaining a complete description of them for the general Cahen-Wallach spacetimes and a large class of examples in the special case of an ε-space. In every case, we also emphasize the description of such hypersurfaces as three-dimensional locally symmetric Brinkmann manifolds. As a byproduct, we determine minimal and CMC examples.

Classification of cohomogeneity-one actions on noncompact symmetric spaces

Ivan Solonenko
(University of Stuttgart, Germany)

The first systematic approach towards classifying isometric cohomogeneity-one actions on symmetric spaces of noncompact type dates back to the work of Berndt and Brück from 2001. Later on, in a series of articles, Berndt and Tamaru reduced the classification problem to an obscure and elaborate problem in the representation theory of reductive Lie groups, known as the nilpotent construction problem. Due to the complexity of that problem, the classification efforts have been at a virtual standstill since 2013. In this talk, I will report on the joint work with Víctor Sanmartín-López, in which we fully resolve the nilpotent construction problem, thus completing the classification of cohomogeneity-one actions on symmetric spaces of noncompact type.

Totally geodesic submanifolds of homogeneous $\mathbb{C}P^{2n+1}$

Andreas Vermeiren
(KU Leuven, Belgium)

In this talk, I will describe the homogeneous metrics on the complex projective spaces $\mathbb{C}P^{2n+1}$ by using two different approaches based on the theory of homogeneous spaces and Riemannian submersions, respectively. Next, we classify totally geodesic submanifolds of dimension at least three in complex projective spaces equipped with an arbitrary homogeneous metric.

This talk is based on an ongoing joint work with Michaël Liefsoens and Alberto Rodríguez Vázquez.

Large automorphism groups of parabolic geometries

Henrik Winther
(UiT - The Arctic University of Norway, Norway)

We discuss maximal and submaximal global symmetry dimensions, i.e. dimensions of automorphism groups of parabolic geometries. This problem in the general context is largely algebraic. If we impose an additional restriction of compactness of the manifold with parabolic geometry, the problem becomes much more complicated.

Joint work with Boris Kruglikov.

Participants

1. Alma Albujer, Universidad de Alicante, Spain
2. Luis Alías, Universidad de Murcia, Spain
3. Efraín Basurto-Arzate, Technische Universität Dortmund, Germany
4. Miguel Brozos Vázquez, Universidade da Coruña, Spain
5. Esther Cabezas-Rivas, Universitat de València, Spain
6. Sandro Caeiro-Oliveira, Universidade de Vigo, Spain
7. Giovanni Calvaruso, Università del Salento, Italy
8. Esteban Calviño Louzao, Consellería de Educación, Spain
9. Salah Chaib, Universidade do Minho, Portugal
10. Ángel Cidre Díaz, Universidade de Santiago de Compostela, Spain
11. Maciej Czarnecki, Uniwersytet ódzki, Poland
12. Manuel de León, ICMAT, Spain
13. Owen Dearricott, La Trobe University, Australia
14. José Carlos Díaz Ramos, Universidade de Santiago de Compostela, Spain
15. Dan Disney, Durham University, United Kingdom
16. Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain
17. Cristina Draper Fontanals, Universidad de Málaga, Spain
18. Manuel Fernández López, Consellería de Educación, Spain
19. Ángel Ferrández, Universidad de Murcia, Spain
20. Ana Ferreira, Universidade do Minho, Portugal
21. María Ferreiro-Subrido, Universidade de Santiago de Compostela, Spain
22. Eduardo García Río, Universidade de Santiago de Compostela, Spain
23. Sandra Gavino, Consellería de Educación, Spain
24. Olga Gil Medrano, U. de Valencia, Spain
25. Ixchel Dzohara Gutiérrez Rodríguez, Universidade de Vigo, Spain
26. Luis Hernández Lamoneda, CIMAT, Mexico
27. Luis M. Hervella Torrón, Universidade de Santiago de Compostela, Spain
28. Jaspreet Kaur, Central University of Punjab, India
29. Filipe Kelmer, Universidad de Almería, Spain
30. Andreas Kollross, University of Stuttgart, Germany
31. Thomas Leistner, University of Adelaide, Australia
32. Juan Manuel Lorenzo Naveiro, Universidade de Santiago de Compostela, Spain
33. Omid Makhmali, UiT - The Arctic University of Norway, Norway
34. Juan Carlos Marrero, Universidad de la Laguna, Spain
35. Dalah Mohamed, University Frères Mentouri Constantine 1, Algeria
36. Rouzbeh Mohseni, Institute of Mathematics, Polish Academy of Sciences, Poland
37. Diego Mojón Álvarez, Universidade de Santiago de Compostela, Spain
38. Marian Ioan Munteanu, University "Al.I.Cuza" of Iasi, Romania
39. Lorenzo Nicolodi, Università di Parma, Italy
40. Tomás Otero Casal, Universidade de Santiago de Compostela, Spain
41. Niklas Otto, Technische Universität Dortmund, Germany
42. Edith Padrón Fernández, Universidad de La Laguna , Spain

43. Vicente Palmer, Universitat Jaume I, Spain
44. Francisco José Palomo, University of Malaga, Spain
45. Lorenzo Pellegrino, Università del Salento, Italy
46. Olga Pérez Barral, Consellería de Educación, Spain
47. Mario Julián Rodríguez Sánchez de Toca, Universidade de Santiago de Compostela, Spain
48. Alberto Rodríguez Vázquez, Université Libre de Bruxelles, Belgium
49. Rosalía Rodríguez-Gigirey Villar, Universidade de Santiago de Compostela, Spain
50. Alfonso Romero, Universidad de Granada, Spain
51. Modesto Salgado Seco, Universidade de Santiago de Compostela, Spain
52. Miguel Sánchez, Universidad de Granada, Spain
53. Adolfo Sánchez-Valenzuela, CIMAT, Mexico
54. Víctor Sanmartín López, Universidade de Santiago de Compostela, Spain
55. Lorenz Schwachhöfer, TU Dortmund University, Germany
56. Paul Schwahn, Université Paris-Saclay, France
57. Ivan Solonenko, University of Stuttgart, Germany
58. Silvia Souto Pérez, Universidade de Santiago de Compostela, Spain
59. Luis Ugarte, Universidad de Zaragoza, Spain
60. M. Elena Vázquez Abal, Universidade de Santiago de Compostela, Spain
61. Ramón Vázquez Lorenzo, Consellería de Educación, Spain
62. Andreas Vermeiren, KU Leuven, Belgium
63. W. Stephen Wilson, Johns Hopkins University, USA
64. Henrik Winther, UiT - The Arctic University of Norway, Norway