Riemannian manifolds not quasi-isometric to leaves in codimension one foliations

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We show that every smooth open manifold of dimension $p \geq 2$ admits a complete Riemannian metric, with bounded geometry (injectivity radius and sectional curvature) and any growth type compatible with bounded geometry, that is not quasi-isometric to a leaf of any codimension one foliation of any closed manifold. This extends the result for $p = 2$ presented at the last International Colloquium at Santiago [2] and answers a question of Attie and Hurder [1] who constructed such metrics on certain manifolds of dimension at least 6. It is shown that every leaf of a codimension one foliation of a compact manifold has a certain "bounded homology property", which states approximately that in the leaf every compact region whose boundary has bounded volume admits a Morse function whose level sets also have bounded (but larger) volume. (Note that on leaves in the interior of a Reeb component the volume of such regions is unbounded, but there are Morse functions whose level sets have bounded volume.) The construction of the Riemannian metrics, starting with a metric of a certain growth type, replaces the metric on an infinite set of disjoint balls of the same small size by "balloons" whose radii are unbounded, as in [2], so that the new metric does not have the bounded homology property. Part of the proof involves a partial generalization of Novikov's theorem on the existence of Reeb components.

References

[1] O. Attie and S. Hurder, Manifolds which cannot be leaves of foliations. *Topology* **35** (1996), 335-353.

[2] P.A. Schweitzer, Surfaces not quasi-isometric to leaves of foliations of compact 3-manifolds. *Analysis and geometry in foliated manifolds*, Proceedings of the VII International Colloquium on Differential Geometry, Santiago de Compostela, 1994. World Scientific, Singapore, (1995), 223-238.