MEAN CURVATURE FLOW OF SPACELIKE GRAPHS

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I am going to talk about my recent joint work with Guanghan Li (arXiv:0804.0783;0801.3850).

We consider the product $\Sigma_1 \times \Sigma_2$ of two Riemannian Manifolds (Σ_i, g_i) with the pseudo-Riemannian metric $g_1 - g_2$, and a map $f : \Sigma_1 \to \Sigma_2$ defining a spacelike graph $F(p) = (p, f(p))$. We assume Σ_1 is compact and Σ_2 complete with bounded curvatures.

We consider the Mean Curvature Flow F_t for $F_0 = F$. Under certain conditions on the sectional curvatures K_i of Σ_i: $K_1 \geq K_2^+$, we prove the flow F_t remains a spacelike graph of a map f_t and exists for all time t, and if Σ_2 is compact then for a sequence $t_n \to +\infty$, F_{t_n} converges at infinity to a maximal graph.

Then we apply Bernstein-Calabi results obtained by L. Alias and A. Albujer (arXiv:0709.4363) for the case $dim(\Sigma_2) = 1$, and extended by the authors (arXiv:0801.3850) to higher codimension $dim(\Sigma_2) = n$, to conclude the limit is the graph of a totally geodesic map and is a slice if $K_1 > 0$ somewhere.

If $K_1 > 0$ everywhere we prove the mean curvature of F_t is exponentially decreasing on t, what allows to drop the compactness assumption and prove the convergence of all the flow to a unique slice.

We apply this result to prove that given any Riemannian manifolds Σ_1, Σ_2, with $K_1 > 0$, for any map $f : \Sigma_1 \to \Sigma_2$ with $f^* g_2 < \rho^{-1} g_1$, f is homotopic to a constant map, where $\rho \geq 0$ is a constant depending only on $sup_{\Sigma_2} K_2$ and on $\min_{\Sigma_1} K_1$. This result largely extends the main result of M-T. Wang in Inventiones Math. 148 (2002), and the particular case $dim(\Sigma_2) = 1$ of the main result in Comm. Pure. Appl. Math 57 (2004) (with M-P. Tsui).

Our methods are simpler then Wang's ones, for we use the pseudo-Riemannian structure of $\Sigma_1 \times \Sigma_2$ instead the Riemannian one used by Wang, and because pseudo-Riemannian geometry has the good signature in the evolution equations, we have better regularity, and therefore we require fewer restrictions on the curvatures and on the map f itself.