Sphere Rigidity in the Euclidean Space

Julien ROTH

The well-known Alexandrov theorem [1] says that embedded hypersurfaces in $\mathbb{R}^{n+1}$ with constant mean curvature are geodesic spheres. This result is not true for only immersed hypersurfaces. For instance, the so-called Wente's tori (see [5]) are examples of compact surfaces with constant mean curvature in $\mathbb{R}^3$, which are not geodesic spheres. Other examples of higher genus are known (see [3] for instance).

For immersed hypersurfaces of constant mean curvature, an additional assumption is needed. One condition is given by the Hopf theorem [2], which says that constant mean curvature spheres immersed in $\mathbb{R}^{n+1}$ are geodesic spheres.

In this talk, we give a new rigidity theorem for spheres, where we replace the topological assumption by a metric assumption. Precisely, it is easy to see that hypersurfaces of $\mathbb{R}^{n+1}$ with constant mean curvature and constant scalar curvature are geodesic spheres. This result comes from the fact that a hypersurface of constant mean curvature and constant scalar curvature is totally umbilic. Here, we give a new rigidity result with a weaker assumption on the scalar curvature. Namely, we show

Théorème 1 *Let (M^n, g) be a compact, connected and oriented Riemannian manifold without boundary isometrically immersed into $\mathbb{R}^{n+1}$. Let h be a positive constant. Then, there exists $\varepsilon > 0$ such that if*

(1) $H = h$ and

(2) $|\mathrm{Scal} - s| \leqslant \varepsilon$,

for some constant s, then M is the sphere $\mathbb{S}^n\left(\frac{1}{h}\right)$ with its standard metric.

We derive this theorem from results about almost umbilic hypersurfaces that we will prove before, and based on a previous eignevalue pinching result given in [4]. We will give a word about this eigenvalue pinching result.

References

[1] A.D. Alexandrov, *Uniqueness theorems for the surfaces in the large I*, Vesnik Leningrad Univ. **11** (1956), 5–17.

[2] H. Hopf, *Differential geometry in the large*, Lecture Notes in Mathematics, vol. 1000, Springer-Verlag, Berlin, 1983.

[3] M. Jleli and F. Pacard, *An end-to-end construction for compact constant mean curvature surfaces*, Pac. J. Math. **221** (2005), no. 1, 81–108.

[4] J. Roth, *Pinching of the first eigenvalue of the Laplacian and almost-Einstein hypersurfaces of Euclidean space*, Ann. Glob. Anal. Geom. (to appear).

[5] H. Wente, *Counterexample to a conjecture of H. Hopf*, Pac. J. Math. **121** (1986), no. 1, 193–243.