ROTATIONAL SURFACES IN H_3 WITH CONSTANT GAUSS CURVATURE

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The Heisenberg's space H_3 is the Riemannian manifold $(\mathbb{R}^3, ds^2)$ with
$$ds^2 = dx^2 + dy^2 + [dz + \tfrac{1}{2}(ydx - xdy)]^2,$$
invariant by rotations about the z-axis. Here one finds a complete description of all rotational surfaces with constant Gauss curvature K in the Heisenberg space H_3. Despite many and substantial similarities with the Euclidean case, this family of surfaces displays some phenomena which do not have their counterpart in $\mathbb{R}^3$. For $K = 0$ there are three cases, for $K > 0$ also three cases, but for $K < 0$ five cases. The profile curves of all these cases are illustrated on the corresponding figures and characterized geometrically.

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