Branching number of a graphed pseudogroup

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There is a way to define an average number of branches per vertex for an infinite locally finite rooted tree. It is the *branching number* introduced by R. Lyons in [2]. This number has an important role in several probabilistic processes such as random walks and percolation.

In this poster, we extend this notion to any measurable graphed pseudogroup Γ of finite type acting on a probability space (X, μ). First, we prove that the branching number $br(x)$ of the orbit $\Gamma(x)$ defines a Borel map. Secondly, we associate the following numbers

$$\underline{br}(\Gamma) = ess\, inf_{x \in X}\, br(x) \qquad br(\Gamma) = \int_X br(x) d\mu(x) \qquad \overline{br}(\Gamma) = ess\, sup_{x \in X}\, br(x)$$

to the graphed pseudogroup Γ. In the ergodic case, they coincide with the branching number of almost every orbit.

Our main theorem asserts that any measurable pseudogroup of finite type acting on a Borel space X with an harmonic measure μ is amenable when its branching number $br(\Gamma)$ is equal to 1. This generalizes former results of C. Series [3] and V. Kaimanovich [1] on equivalence relations with polynomial and subexponential growth. To prove it, we construct an example of minimal lamination with exponential growth whose holonomy pseudogroup has branching number equal to 1.

References

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