# Completely integrable embeddings in open manifolds

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Let $M$ be an open, oriented smooth $n$-manifold, and $L$ an oriented smooth $k$-manifold ($k < n$), possibly noncompact and disconnected. We say that a (proper) embedding $h : L \to M$ is

a) *completely integrable* (CI) if there exists a smooth submersion $\Phi : M \to \mathbb{R}^m$, $m := n - k$, such that $h(L) \subset \Phi^{-1}(0)$,
b) *strongly completely integrable* (SCI) if $\Phi^{-1}(0) = h(L)$.

The aim of this talk is to present some results recently obtained by the authors concerning the following problems:

(i) which manifolds admit CI (or SCI) embeddings in a given ambient manifold $M$? specially in $M = \mathbb{R}^n$?
(ii) does this CI or SCI character depend on the particular embedding of $L$? or only on $L$ or on the dimension of $M$?
(iii) since CI embeddings give rise to submanifolds $h(L) \subset M$ with trivial normal bundle, will any submanifold with trivial normal bundle in $M$ be CI or SCI?

On the other hand, for any completely integrable embedding $h$, the submanifold $h(L)$ is a leaf of the *simple foliation* defined by the level "surfaces" of $\Phi$. Thus our problems include the question of constructing foliations in open manifolds with prescribed leaves. This is particularly relevant when we assume that $M$ is diffeomorphic to $\mathbb{R}^n$; for example we show that no sphere $\mathbb{S}^k$ is leaf of a $k$-dimensional foliation in $\mathbb{R}^n$ ($n \leq 2k$), result highly non trivial for $k = 3, 7$, and that any embedding of $\mathbb{S}^1$ in $\mathbb{R}^n$, $n \geq 3$, is SCI.

Our proofs rely on a relative version of the Phillips-Gromov h-principle, obstruction theory and several results from the theory of immersions. Our results include the corresponding earlier results for links in $\mathbb{R}^3$ due to Watanabe and Miyoshi.