BAR BILLIARDS AND PONCELET'S PORISM

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Let us consider two ovals C_1 and C_2, i.e., two smooth strictly convex closed plane curves. Let the oval C_2 lie inside second oval C_1. From any point on C_1, draw a tangent to C_2 and extend it to C_1 in the opposite direction. From this point we draw another tangent, etc. For all tangents, the resulting Poncelet's transverse will be called a bar billiard since it is similar to some traditional game played years ago. In general in this game players scored points by knocking balls into the holes while avoiding to topple a skittle in the middle of the table. Here the role of the skittle plays the oval C_2 and it must be "toppled" by the tangent line. We will concentrate on the Poncelet's porism in this specific setting and we say that a bar billiard has a Poncelet's porism property if the following is true: if, on the oval C_1, there is one point of origin for which a Poncelet transverse is closed, then the adequate transverse will also close for any other point of origin on the oval. In this talk we prove that for a given oval C there exist ovals C_{in} and C_{out}, inside and outside of C, such that the pairs (C, C_{in}) and (C_{out}, C) satisfy the Poncelet's porism for almost any number of reflections in their bar billiards.

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