ON THE CLASSIFICATION OF BIHARMONIC CUVES IN $\mathbb{C}P^2$

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In [8], even if they took the main interest in harmonic maps, Eells and Sampson also envisaged some generalizations and defined *biharmonic* maps $\varphi : (M, g) \to (N, h)$ between Riemannian manifolds as critical points of the *bienergy* functional $E_2(\varphi) = \frac{1}{2}\int_M |\tau(\varphi)|^2 \, v_g$, where $\tau(\varphi)$ is the tension field of φ. Biharmonic maps are a natural extension of harmonic maps ($\tau(\varphi) = 0$) and they are solutions of the biharmonic equation:

$$\begin{aligned} \tau_2(\phi) &= -J(\tau(\phi)) = -\Delta\tau(\phi) - \operatorname{trace} R^N(d\phi, \tau(\phi))d\phi \\ &= 0, \end{aligned}$$

where J is the Jacobi operator of ϕ, Δ is the rough Laplacian defined on section of $\phi^{-1}(TN)$.

Although E_2 has been on the mathematical scene since the early '60 (when some of its analytical aspects have been discussed) and regularity of its critical points is nowadays a well-developed field, a systematic study of the geometry of biharmonic maps has started only recently.

If $\varphi : I \subset \mathbb{R} \to (N, h)$ is a curve the biharmonic equation reduces to the fourth order differential equation

$$\nabla^3_{\varphi'}\varphi' - R^N(\varphi', \nabla_{\varphi'}\varphi')\varphi' = 0.$$

This equation has been studied intensively in the last decade (see, for examples, [1, 2, 3, 4, 5, 6, 7]) and several constructions and classifications of biharmonic curves have been obtained. In particular, proper biharmonic curves are classified in: surfaces, space forms and three dimensional homogeneous spaces.

In this lecture we shall focus our attention on biharmonic curves on $\mathbb{C}P^2$ and we shall give the complete classification of these curves.

We shall relate the notion of proper biharmonic curves of $\mathbb{C}P^n$ with that of holomorphic helices, that is curves with constant complex torsions in the sense of S. Maeda and Y. Ohnita [9].

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