On the violation of Thurston's inequality for codimension 1 foliations

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This is a report on a joint work with Hiroki Kodama, Atsuhide Mori, and Shigeaki Miyoshi on the violation of Thurston's absolute inequality for codimension one foliations.

For an oriented foliation $\mathcal{F}$ of codimension 1 on an oriented 3-manifold M, Thurston's inequality has two versions, namely the absolute one for an embedded closed oriented surface and the relative one for an orieneted compact surface with (positively) transverse boundary. As Thurston proved that if for a foliation on a closed manifold without Reeb components, both inequalities hold. Thus we focus our interests on foliations with Reeb components, in particular, first of all, those who is associated with a spinnable structure of M. Here 'spinnable structure' is a synonim of 'open book decomposition'. There are many of them who satisfy the inequlities.

1. We give a sufficient condition for the violation of the absolute inequality in terms of the monodromy of the spinnable structure. This is related to sobering arcs.

The analoguous inequalities for contact structures are known as Thurston-Bennequin's inequaslities. With a spinnable structure, an isotopic family of contact structures is associated by so called Thurston-Winkelnkemper's construction, which converges to the foliation associated with the spinnable structure.

For contact structures, the relative inequality, whose approval is equiavent to the tightness, implies the absolute inequality. In general, for foliations two inequalities are logically independent. However, there seems to exist very few foliations who satisfy the relative one and still break the absolute one.

2. We explain that the above convergence is so good that through it we can show that relative Thurston's inequality implies the absolute one for spinnable foliations.

There exist other foliations with Reeb components than spinnable ones which satisfies absolute Thurston's inequality.

3. Together with the basic works concerning above 1, Mori and Miyoshi gave a systematic productions of such foliations, relying on the Dehn filling argument.

These results suggest us to concider further problems. For example, what can we claim about the convergence of contact structures associated with Dehn fillings of general coeficients?

Also higher dimensional case is an interesting question in future. In recent years, many attempts to formulate 'overtwistedness' of contact structure in higher dimension have been made and some of them are successfull to a certain degree. A. Mori's formulation suggests some possibility to generalize our previous study to higher dimensional case.

If the time and the situation allow, we would like to discuss the above problems.

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