DISTRIBUTIONS ON THE COTANGENT BUNDLE FROM TORSION-FREE CONNECTIONS

J. Kurek and W. M. Mikulski

Summary: Let m and q be arbitrary integers such that $m \geq 1$ and $0 \leq q \leq 2m$. We study the problem how a torsion-free classical linear connection ∇ on an m-dimensional manifold M induces canonically a smooth (C^∞) distribution $A(\nabla) \subset TT^*M$ on the cotangent bundle T^*M of M such that $dim(A(\nabla)_\omega) = q$ for any $\omega \in T^*M$. We have the following simple examples:

(a) The zero distribution $A^{[1]}(\nabla)$ such that $A^{[1]}(\nabla)_\omega = \{0\}$ for any $\omega \in T^*M$;

(b) The vertical distribution $A^{[2]}(\nabla) = VT^*M$;

(c) The full distribution $A^{[3]}(\nabla) = TT^*M$;

(d) The ∇^*-horizontal distribution $A^{[0]}(\nabla) = H^{\nabla^*}$ on T^*M (i.e. the horizontal distribution of the linear general connection ∇^* on $T^*M \to M$ dual to ∇).

We have the decomposition $TT^*M = VT^*M \oplus H^{\nabla^*}$. We recall that given $v \in T_xM$ and $\omega \in (T^*M)_x$, $x \in M$, we have the ∇^*-horizontal lift $v^{\nabla^*}_\omega$ of v at ω, i.e. the unique vector from $H^{\nabla^*}_\omega$ over v with respect to the cotangent bundle projection.

We have the following family of modifications of H^{∇^*};

(e) Given a canonically dependent on ∇ fibred map $B(\nabla) : T^*M \to T^*M \otimes T^*M$ covering id_M, we have the $B(\nabla)$-modification $A^{[B]}(\nabla)$ of H^{∇^*} such that $A^{[B]}(\nabla)_\omega := \{v^{\nabla^*}_\omega + \frac{d}{dt}_{|t=0}(\omega + t < B(\nabla)(\omega), v >) \mid v \in T_xM\}$ for any $\omega \in (T^*M)_x$, $x \in M$. Then $A^{[B]}(\nabla)$ is a smooth distribution on T^*M of dimension m at any point. Clearly, if $B(\nabla) = 0$ then $A^{[0]}(\nabla) = H^{\nabla^*}$.

We see that $dim(A^{[1]}(\nabla)_\omega) = 0$, $dim(A^{[2]}(\nabla)_\omega) = m$, $dim(A^{[3]}(\nabla)_\omega) = 2m$ and $dim(A^{[B]}(\nabla)_\omega) = m$ for any $\omega \in T^*M$.

The main result of the present note can be roughly formulated as follows.

Theorem A. *All smooth distributions $A(\nabla) \subset TT^*M$ on T^*M with $dim(A(\nabla)_\omega) = const$ canonically depending on a torsion-free classical linear connection ∇ on M are the mentioned above distributions $A^{[1]}(\nabla)$, $A^{[2]}(\nabla)$, $A^{[3]}(\nabla)$ and $A^{[B]}(\nabla)$ for fibred maps $B(\nabla) : T^*M \to T^*M \otimes T^*M$ covering id_M.*

Remark A. One can show (using method from [1]) that the vector space (over **R**) of all $B(\nabla) : T^*M \to T^*M \otimes T^*M$ in question is 3-

dimensional and the fibred maps $B^{[i]}(\nabla) : T^*M \to T^*M \otimes T^*M$ given by $B^{[1]}(\nabla)(\omega) = \omega \otimes \omega$, $B^{[2]}(\nabla)(\omega) = sym((Ric_\nabla)_x)$ and $B^{[3]}(\nabla)(\omega) = alt((Ric_\nabla)_x)$ for $\omega \in (T^*M)_x$, $x \in M$, form the basis in this vector space.

References

[1] Kolář I., Michor P.W., Slovák J., Natural Operations in Differential Geometry, Springer-Verlag Berlin 1993.

[2] Paluszny M., Zajtz A., Foundations of the geometry of natural bundles, Lect. Notes Univ. Caracas, 1984.

Institute of Mathematics
Maria Curie-Sklodowska University
pl. M. Curie-Sklodowska 1, Lublin (Poland)
e-mail: kurek@hektor.umcs.lublin.pl

Institute of Mathematics
Jagiellonian University
Reymonta 4, Kraków, Poland
e-mail: : Wlodzimierz.Mikulski@im.uj.edu.pl