DUALITY AND MINIMALITY FOR RIEMANNIAN FOLIATIONS ON OPEN MANIFOLDS

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Let \mathcal{F} be a Riemannian foliation of dimension l and codimension k on a smooth manifold M. If M and \mathcal{F} are orientable and M is closed, the de Rham spectral sequence of \mathcal{F} is finite dimensional and verifies the duality condition

$$E_2^{s,t} \cong E_2^{k-s,l-t}$$

Moreover, the manifold admits a metric such that each leaf is minimal if and only if the basic cohomology in maximum degree, $E_2^{k,0}$, is non null [1].

When M is an open manifold and the closures of the leaves define a regular compact foliation, a similar result holds,

$$E_2^{s,t} \cong E_{2,c}^{k-s,l-t},$$

where $E_{2,c}$ denote the spectral sequence with compact supports. The finiteness requires a natural additional condition. And the foliation is minimal if and only if $E_{2,c}^{k,0} \neq 0$.

A singular Riemannian foliation on a compact manifold induces, on the regular stratum, a foliation with the good properties. This case was considered in [2], where they prove, by a different method, the above minimality characterisation.

References

- Masa, X. Duality and minimality in riemannian foliations, Comment. Math. Helv. 67 (1992), 17–27.
- [2] Saralegi-Aranguren, M., Royo Prieto, J.I. and Wolak, R. Tautness for riemannian foliations on non-compact manifolds, Manuscripta Math. 126 (2008), 177-200