# BIHARMONIC SUBMANIFOLDS IN NON-SASAKIAN CONTACT METRIC 3-MANIFOLDS

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## Abstract

A contact metric manifold whose characteristic vector field is a harmonic vector field is called *H-contact metric manifold*. We introduce the notion of $(\kappa, \mu, \nu)$*-contact metric manifolds* in terms of a specific curvature condition, where $\kappa, \mu, \nu$ are smooth functions. Then, we prove that a contact metric 3-manifold $M(\eta, \xi, \phi, g)$ is an $H$-contact metric manifold if and only if it is a $(\kappa, \mu, \nu)$-contact metric manifold on an everywhere open and dense subset of $M$. Moreover, it is proved that in dimension greater than three such manifolds are reduced to $(\kappa, \mu)$-contact metric manifolds. On the contrary, for the dimension three such $(\kappa, \mu, \nu)$-contact metric manifolds exist.

In the study of contact manifolds, Legendre curves play an important role, since a diffeomorphism of a contact manifold is a contact transformation if and only if it maps Legendre curves to Legendre curves. We prove that biharmonic Legendre curves in 3-dimensional $(\kappa, \mu, \nu)$-contact metric manifolds are necessarily geodesics. Moreover, we give examples of Legendre geodesics in these spaces. We also give a nice geometric interpretation of 3-dimensional $(\kappa, \mu, 0)$-contact metric manifolds in terms of its Legendre curves. Next, we study non-minimal biharmonic anti-invariant surfaces of 3-dimensional $(\kappa, \mu, \nu)$-contact metric manifolds. Especially, for the 3-dimensional $(\kappa, \mu, 0)$-contact metric manifolds, we prove that biharmonic anti-invariant surfaces, with constant norm of the mean curvature vector field, are minimal. Furthermore, we give an example of an anti-invariant surface with constant norm of the mean curvature vector field, immersed in these spaces.

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