Transversely Cantor laminations as inverse limits*

Álvaro Lozano Rojo

Transversely Cantor laminations appears naturally in the study of foliations as exceptional minimal sets, and are also releated with group actions on the Cantor set. In order to construct examples, we take a manifold M, a submersion $f : M \to M$ (which is not a diffeomorphism) and the inverse limit

$$X = \varprojlim(M, f) = \left\{ (m_k) \in \prod_{k \geq 0} M \;\middle|\; m_k = f(m_{k+1}) \right\}.$$

It is a foliated fibre bundle over M with fibre the Cantor set. R.F. Williams has shown a broader family of examples in [4]: he replaces the manifold M with a Riemannian branched manifold and f by a branched immersion. Moreover, he proves that any expanding attractor of a diffeomorphism of a manifold is homeomorphic to such a inverse limit [4]. In the same way, J. Bellisard, R. Benedetti and J.-M. Gambaudo have proved that the continuous *hull* of an aperiodic and repetitive tiling of the plane is a inverse limit of branched flat surfaces [1]. In fact, this result hold in a more general setting where continuous hulls is replaced by $\mathbb{G}$-solenoids [2], i.e. transversely Cantor laminations defined by free actions of a Lie group $\mathbb{G}$. Both results generalises a former one of A. Vershik which describes minimal dynamical systems on the Cantor set as inverse limits of graphs.

The aim of this talk is to show that these inverse limits are actually the only examples of minimal transversely Cantor laminations without holonomy:

Theorem. *Any transversely Cantor minimal lamination without holonomy* $(M, \mathcal{L})$ *is a inverse limit of a sequence branched manifolds* S_n *and immersions* $f_n : S_n \to S_{n-1}$ *between them.*

In the oriented and transversely oriented case, using the same ideas of [1, 2] for $\mathbb{G}$-solenoids, we identify the space of positive measures invariants by the action of the holonomy grupoid with $\varprojlim\big(H_p^+(S_n; \mathbb{R}), f_n^*\big)$, with $p = \dim \mathcal{L}$.

References

[1] J. Bellissard, R. Benedetti, and J.-M. Gambaudo, *Spaces of tilings, finite telescopic approximations and gap-labelling*, Comm. Math. Phys., 261 (2006), 1–41.

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[2] R. Benedetti and J.-M. Gambaudo, *On the dynamics of $\mathbb{G}$-solenoids. Applications to Delone sets*, Ergodic Theory Dynam. Systems, 23 (2003), pp. 673–691.

[3] Á. Lozano Rojo, *Dinámica transversa de laminaciones definidas por grafos repetitivos*, PhD thesis, Universidad del País Vasco/Euskal Herriko Unibertsitatea, 2008.

[4] R. F. Williams, *Expanding attractors*, Publications mathématiques de l'I.H.É.S., 43 (1974), 169–203.