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Proposal

I would like to propose to give a talk with the title

Reconstructing the geometric structure of a Riemannian symmetric space from its Satake diagram.

Abstract. The (local) geometry of a Riemannian symmetric space is completely controlled by its curvature tensor. For example, the totally geodesic submanifolds of such a space are in one-to-one correspondence with its Lie triple systems, i.e. with the curvature-invariant linear subspaces of its tangent space. For this reason it is desirable to have a representation of the curvature tensor available for study for every Riemannian symmetric space $M = G/K$. In particular, it is very desirable to have a representation which is also suited for computer-based calculations with a computer algebra package like, for example, Maple.

The well-known formula $R(u,v)w = -[[u,v],w]$ relating the curvature tensor R of M to the Lie bracket of the Lie algebra $\mathfrak{g}$ of the transvection group G lets one calculate R relatively easily if G is a classical group (then $\mathfrak{g}$ is a matrix Lie algebra, and its bracket simply the commutator of matrices), but not so easily if G is one of the exceptional Lie groups, because then the explicit description of $\mathfrak{g}$ as a matrix algebra is too unwieldy to be useful generally.

In the talk I will therefore describe another approach, which permits the reconstruction of the Riemannian symmetric space from its Satake diagram (which is the Dynkin diagram of the Lie algebra $\mathfrak{g}$, "enhanced" by additional information describing the involution $\sigma : \mathfrak{g} \to \mathfrak{g}$ which determines the symmetric structure of M). It is known from the general theory that the Satake diagram determines the symmetric space up to local isometry, and therefore in particular the curvature tensor. However, it turns out that to actually obtain a description of the curvature tensor which is explicit enough for calculations, some new work needs to be invested.

In my talk, I will describe the algorithms which accomplish the reconstruction of the curvature tensor from the Satake diagram, involving the root space decomposition of $\mathfrak{g}$, its Chevalley/Weyl structure constants, and a description of the involution σ. I will also discuss my implementation of these algorithms for Maple. The algorithms are also found in my paper `arXiv:0801.4127` (submitted for publication); the Maple implementation can be downloaded from `http://satake.sourceforge.net` .