KÄHLER MANIFOLDS WITH QUASI-CONSTANT HOLOMORPHIC CURVATURE.

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0. Abstract. The aim of the present paper is to classify compact, simply connected Kähler manifolds (M,g,J) admitting global, 2-dimensional, J-invariant distribution $\mathcal{D}$ satisfying the following property: The holomorphic curvature $K(\pi)=R(X,JX,JX,X)$ of any J-invariant 2-plane $\pi\subset T_xM$, where $X\in\pi$ and $g(X,X)=1$ depends only on the point x and the number $|X_{\mathcal{D}}|=\sqrt{g(X_{\mathcal{D}},X_{\mathcal{D}})}$, where $X_{\mathcal{D}}$ is an orthogonal projection of X on $\mathcal{D}$. In this case we have

$$R(X,JX,JX,X)=\phi(x,|X_{\mathcal{D}}|)$$

where $\phi(x,t)=a(x)+b(x)t^2+c(x)t^4$ and a,b,c are smooth functions on M. Also $R=a\Pi+b\Phi+c\Psi$ for certain curvature tensors $\Pi,\Phi,\Psi\in\bigotimes^4\mathfrak{X}^*(M)$ of Kähler type. The investigation of such manifolds, called QCH manifolds, was started by G. Ganchev and V. Mihova. In our paper we shall use their local results to obtain a global classification of such manifolds under the assumption that $\dim M=2n\geq 6$. By $\mathcal{E}$ we shall denote the $(\dim M-2)$-dimensional distribution which is an orthogonal complement of $\mathcal{D}$ in TM. If $\{X,JX\}$ is any local orthonormal basis of $\mathcal{D}$ then the function $\kappa=\sqrt{(div_{\mathcal{E}}X)^2+(div_{\mathcal{E}}JX)^2}$ does not depend on the choice of X,JX. We classify QCH compact, simply connected Kähler manifolds satisfying the conditions int $B=\emptyset$ and $U\neq\emptyset$ where $B=\{x\in U: b(x)=0\}, U=\{x\in M:\kappa(x)\neq 0\}$. First we shall show that (M,g,J) admits a global holomorphic Killing vector field with a Killing potential, which is a special Kähler-Ricci potential. Next we use the results of Derdzinski and Mashler, who classified compact Kähler manifolds admitting special Kähler-Ricci potentials. As a corollary we prove that the only compact, simply connected QCH manifold with $\kappa\neq 0$ and analytic Riemannian metric g is a holomorphic $\mathbb{CP}^1$-bundle over $\mathbb{CP}^{n-1}$ (with $\mathcal{D}$ being an integrable distribution whose leaves are the fibers $\mathbb{CP}^1$ of the bundle) or is $\mathbb{CP}^n$ with metric of constant holomorphic sectional curvature (in this case $\mathcal{D}$ is any J-invariant 2-dimensional distribution on $\mathbb{CP}^n$ with $\kappa\neq 0$, however such distributions may not exist).