Foliations and $(k, \mu, \nu)-$nullity

Adrian Mihai Ionescu

We introduce a $(k, \mu, \nu)-$nullity condition for the Riemannian curvature tensor on $f-p.k.-$manifolds, namely

$$R(A,B)\xi_j = \sum_i k_{ij}(\eta_i(A)\varphi^2(B) - \eta_i(B)\varphi^2(A))$$
$$+ \sum_i \mu_{ij}(\eta_i(B)h_i(A) - \eta_i(A)h_i(B))$$
$$+ \sum_i \nu_{ij}(\eta_i(B)\varphi h_i(A) - \eta_i(A)\varphi h_i(B))$$

where $h_i = -\frac{1}{2}L_{\xi_i}\varphi$ and $k_{ij}, \mu_{ij}, \nu_{ij}$ are smooth functions.

We study the consequences of this type conditions on the geometry of the manifold; in particular integrable distributions(foliations) will become apparent. The characteristic foliation (spanned by the globally defined vector fields ξs) is totally geodesic with flat leaves; the complementary distribution D supports sub-foliations given by the eigenspaces of the (self-adjoint) *pseudo-characteristic* tensor fields $m_{ij} = \mu_{ij}h_i + \nu_{ij}\varphi h_i$. If the manifold is an almost $C-$manifold, then D is integrable and has Kaehler leaves.