

Paraquaternionic manifolds and mixed 3-structures

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Abstract

The paraquaternionic structures, firstly named quaternionic structures of second kind, have been introduced in geometry by P. Libermann, in 1952 [C.R. Acad. Sc. Paris 234 (1952)]. The theory of paraquaternionic manifolds parallels the theory of quaternionic manifolds, but uses the algebra of paraquaternionic numbers, in which two generators have square 1 and one generator has square -1. Accordingly, such manifolds are equipped with a subbundle of rank 3 in the bundle of the endomorphisms, locally spanned by two almost product structures and one almost complex structure. From the metric point of view, the almost paraquaternionic Hermitian manifolds have neutral signature.

The counterpart in odd dimension of paraquaternionic geometry was introduced in by the present authors and R. Mazzocco [Mediterranean J. Math. 3 (3-4) (2006)]. It is called mixed 3-structure, which appears in a natural way on lightlike hypersurfaces in paraquaternionic manifolds. We give examples of manifolds endowed with mixed 3-structures and obtain some properties. Particularly, we obtain that a compatible metric with a mixed 3-structures is necessarily semi-Riemann and mixed 3-Sasakian manifolds are Einstein, hence the possible importance of these structures in theoretical physics.