Transversal Killing spinors on Riemannian flows

(joint work with Nicolas Ginoux)

Killing spinors on Riemannian spin manifolds are smooth sections of the spinor bundle of which covariant derivative is proportional to the Clifford multiplication. Those manifolds carrying non-zero Killing spinors have been well understood for a long time. Such manifolds are Einstein. In particular, the existence of non-zero Killing spinors imposes very rigid conditions to the geometry of the underlying manifold.

In this talk we transpose the Killing spinor equation to the set up of spin Riemannian flows. In the definition of that equation - that we call transversal Killing spinor - we allow the first derivatives of the spinor field to behave differently along the leaves and along the orthogonal distribution of the flow respectively. First, we describe important examples of Riemannian flows carrying transversal Killing spinors, which arise as submersions over manifolds with Killing spinors. We then study integrability conditions on Sasakian manifolds and we show that they can be related to the classical ones. In particular, we describe these equations on Berger spheres. Finally, we end with the case of 3-dimensional Riemannian flows. We illustrate them on Bieberbach manifolds and we classify all compact minimal flows carrying such spinors.