Geometric realizations of algebraic curvature tensors

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Let V be a finite m-dimensional vector space for $m \geq 3$. We say that $R \in V^* \otimes V^* \otimes End(V)$ is an algebraic curvature operator if it has the symmetries of a torsion free connection on the tangent bundle of some manifold, i.e. if $A(x, y) = -A(y, x)$ and $A(x, y)z + A(y, z)x + A(z, x)y$. We show that any algebraic curvature operator is geometrically realizable by a torsion free connection ∇ on the tangent bundle of V. We also discuss other similar geometrical realization theorems showing, for example, that any Ricci flat (resp. Ricci symmetric or Ricci antisymmetric) algebraic curvature operator is geometrically realizable by a torsion free Ricci flat (resp. Ricci symmetric or Ricci antisymmetric) connection on $T(V)$. We also establish similar geometrical realization theorems which arise from considering the decomposition of the space of algebraic curvature operators with respect to the natural action of the general linear group $GL(V)$. This is joint work with Stana Nikcevic and Dan Westerman.