Cohomogeneity one isometric actions on quaternionic hyperbolic spaces

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A cohomogeneity one action on a Riemannian manifold is a proper isometric action with codimension one orbits or, in other words, with one-dimensional orbit space. Their investigation in specific ambient manifolds is linked to interesting geometric features of the ambient space, as well as to algebraic properties of its isometry group. By the very definition, cohomogeneity one actions only exist in ambient manifolds with large isometry group, such as homogeneous spaces. Indeed, their investigation in the subclass of symmetric spaces constitutes a problem that traces back to the 30s.

Cohomogeneity one actions on Riemannian symmetric spaces of rank one (that is, spheres, and projective and hyperbolic spaces over the four real division algebras) have been classified up to orbit equivalence, except on quaternionic hyperbolic spaces. In this talk I will report on a joint work with J. Carlos Díaz-Ramos and Alberto Rodríguez-Vázquez where we conclude the classification of cohomogeneity one actions on symmetric spaces of rank one. The proof is linked to a seemingly difficult problem of linear algebra in quaternionic Euclidean spaces, for which we develop a combination of geometric, topological and algebraic ideas.