The geometry of group presentations and the winding invariant

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A presentation P of a group G can be transformed into another presentation P' of G by performing certain changes in the set of relators, called Q^* -transformations. In this case P and P' have homotopy equivalent standard complexes. Moreover, they are simple homotopy equivalent. We will construct presentations P and P' with simple homotopy equivalent complexes and which are not Q^* -equivalent. These complexes are not contractible. An example with contractible standard complexes would be a counterexample to the Andrews-Curtis conjecture, open since 1965. In this talk we will introduce a new invariant which associates a Laurent polynomial in two variables to every element in the commutator subgroup of the free group of rank 2. This invariant is one of the key ideas to construct the presentations P and P'.