Some homogeneous Lagrangian submanifolds in complex hyperbolic spaces

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  - A submfd L in a symplectic mfd (M, ω) with $\omega|_L = 0$ & $\mathrm{dim} L = \frac{1}{2}\mathrm{dim} M$.
  - A widely-studied class of higher codimentional submfds by motivations related to Riemannian & Symplectic geometry.
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- Problem: *Construct and classify homogeneous Lagrangian submanifolds in a specific Kähler manifold (e.g. Hermitian symmetric spaces).*

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e.g. T^n-orbits in a toric Kähler manifold, real forms in cplx flag mfds, Gauss images in $\tilde{G}r_2(\mathbb{R}^{n+2})$ of homog. hypersurfaces in a sphere... etc.

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Classification results (of actions admitting Lag orbits):

- $M = \mathbb{C}P^n$ & H is a cpt simple Lie group [Bedulli-Gori 08]. (Note that $\exists$ 1-1 correspondence btw cpt homog Lag in $\mathbb{C}P^n$ and the ones in $\mathbb{C}^{n+1}$ via Hopf fibration).
- $M = \tilde{G}r_2(\mathbb{R}^{n+2}) \simeq Q_n(\mathbb{C})$ [Ma-Ohnita 09]

Note: so far, we do not know any comprehensive method to classify homog Lag even for Hermitian symmetric spaces...

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Theorem (cf. McDuff 88, Deltour 13)

Let $M = G/K$ be a Hermitian symmetric space of non-compact type, and $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ the Cartan decomposition. Then, there exists a K-equivariant symplectic diffeomorphism $\Phi : (M, \omega) \to (\mathfrak{p}, \omega_o)$.

(Remark: this result is just an existence theorem, although they proved for more general setting [McDuff 88, Deltour 13])

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e.g. $M = \mathbb{C}H^n \simeq B^n$.

$$\Phi : B^n \to \mathbb{C}^n \simeq \mathfrak{p}, \quad z \mapsto \sqrt{\frac{1}{1 - |z|^2}} \cdot z$$

is a K-equivariant symplectic diffeomorphism (not holomorphic).
(Remark: [Di Scala-Loi 08] gives an explicit construction of Φ for any Hermitian symmetric space of non-cpt type.)

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(continued) Since $\Phi : M \to \mathfrak{p}$ is K-equivariant and K is a maximal compact subgroup of G, $\exists$ a map

$$\{\text{cpt homog Lag in } M = G/K\} \longrightarrow \{\text{cpt homog Lag in } \mathfrak{p} \simeq \mathbb{C}^n\}.$$

In this sense, the classification problem of cpt homog Lag in M is reduced to find an $H \subset \mathrm{Ad}(K)$ admitting a Lag orbit in $\mathfrak{p} \simeq \mathbb{C}^n$.

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For example, if M is rank 1, we see $\mathrm{Ad}(K) = U(n)$, and it turns out that

Theorem (Hashinaga-K. 17, Ohnita)

Suppose $M = \mathbb{C}H^n$ and let L' be any cpt homog Lag in $\mathfrak{p} \simeq \mathbb{C}^n$. Then, $L := \Phi^{-1}(L')$ is a cpt homog Lag in $\mathbb{C}H^n$. In particular, any cpt homog Lag in $\mathbb{C}H^n$ (up to congruence) is obtained in this way.

A geometric interpretation:

$$\begin{array}{ccccccc}
 & & (\mathbb{C}H^n, \omega) & \xrightarrow[\text{symp. diffeo.}]{\Phi} & (\mathbb{C}^n, \omega_0) \\
 & & \cup & & \cup \\
C(K) = S^1 \curvearrowright L & \to & S_r^{2n-1} & \xrightarrow[\text{diffeo.}]{\Phi} & S^{2n-1}(\sinh r) \\
\downarrow & & S^1 \downarrow & & S^1 \downarrow \\
L/S^1 & \to & \mathbb{C}P^{n-1}(\frac{4}{\sinh^2 r}) & = & \mathbb{C}P^{n-1}(\frac{4}{\sinh^2 r})
\end{array}$$

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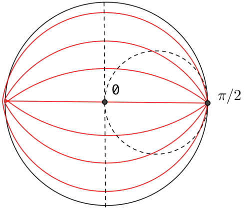
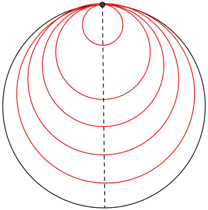


L_0-orbits (A-orbits)



$L_{\pi/2}$-orbits (N-orbits)

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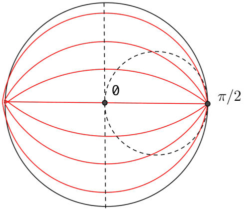

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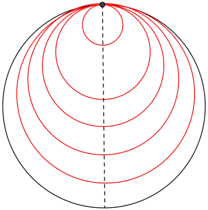

$L_{\pi/2}$-orbits (N-orbits)

(Note: Since $\Phi : M \to \mathfrak{p}$ is a symplectic diffeo, we have a correspondence

$$\{\text{Lag submfd in HSS of non-cpt type } M\} \longleftrightarrow \{\text{Lag submfd in } \mathfrak{p} \simeq \mathbb{C}^n\}.$$

Thus, a construction of (homog) Lag submfd in M provides a way of constructing (new example of) a Lag submfd in $\mathbb{C}^n$.)

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- $\mathfrak{a} \subset \mathfrak{p}$: a maximal abelian subspace of $\mathfrak{p}$.
- $\mathfrak{g} = \mathfrak{g}_0 + \sum_{\lambda \in \Sigma} \mathfrak{g}_\lambda$: the restricted root decomposition w.r.t. $\mathfrak{a}$.
- Letting $\mathfrak{n} := \sum_{\lambda \in \Sigma_+} \mathfrak{g}_\lambda$, we obtain the *Iwasawa decomposition*

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{a} \oplus \mathfrak{n},$$

and $\mathfrak{s} := \mathfrak{a} + \mathfrak{n}$ is so called the solvable part of the Iwasawa decomposition.

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Fact Let S be a connected subgroup of G whose Lie algebra is $\mathfrak{s}$. Then, *S acts on M simply transitively.*
Hence, we obtain an identification $M \simeq S$ (as a Kähler mfd), and this is so called the solvable model of M.

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Since S acts on M simply transitively, the classification of non-cpt homog Lag in M obtained by a subgroup S' of S is reduced to classify Lagrangian subalgebras of $\mathfrak{s}$, that is, Lie subalgebra $\mathfrak{l}$ of $\mathfrak{s}$ satisfying Lagrangian condition i.e., $\omega|_{\mathfrak{l}} = 0$ and $\dim \mathfrak{l} = \frac{1}{2} \dim \mathfrak{s}$.

In [Hashinga-K. 17], we completely classify the Lagrangian subalgebra of $\mathfrak{s}$ when $M = \mathbb{C}H^n$, and give the details of Lagrangian orbits.

Non-cpt homg Lag in $\mathbb{C}H^n$

(The construction) Assume $M = \mathbb{C}H^n$. Then

$$\mathfrak{s} = \mathfrak{a} \oplus \mathfrak{g}_\alpha \oplus \mathfrak{g}_{2\alpha} = (\mathfrak{a} \oplus \mathfrak{g}_{2\alpha}) \oplus \mathfrak{g}_\alpha.$$

Both subspaces $\mathfrak{a} \oplus \mathfrak{g}_{2\alpha}$ and $\mathfrak{g}_\alpha$ are symplectic (complex) subspace of $\dim_{\mathbb{C}}(\mathfrak{a} \oplus \mathfrak{g}_{2\alpha}) = 1$ and $\dim_{\mathbb{C}} \mathfrak{g}_\alpha = n - 1$, hence, taking Lagrangian subspaces $\mathfrak{l}_1 \subset (\mathfrak{a} \oplus \mathfrak{g}_{2\alpha})$ and $\mathfrak{l}_2 \subset \mathfrak{g}_\alpha$,

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For $X, Y \in \mathfrak{l}_1 \subset \mathfrak{a} \oplus \mathfrak{g}_{2\alpha}$ and $U, V \in \mathfrak{l}_2 \subset \mathfrak{g}_\alpha$, the bracket relation of $\mathfrak{s}$ implies

$$\begin{aligned}[X + U, Y + V] &= c_1 U + c_2 V + \{\omega_{\mathfrak{s}}(X, Y) + \omega_{\mathfrak{s}}(U, V)\} Z \\ &= c_1 U + c_2 V \in \mathfrak{l}_2\end{aligned}$$

for some c_1, c_2. Hence, $\mathfrak{l}$ is a subalgebra of $\mathfrak{s}$.
(Remark: This construction is partially generalized to higher rank case [Hashinaga 18])

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(continued) Conversely, we proved the following:

Lemma (H-K)

Let $\mathfrak{s}'$ be any Lagrangian subalgebra of $\mathfrak{s}$. Then, $\mathfrak{s}'$ splits into a direct sum $\mathfrak{s}' = \mathfrak{l}_1 \oplus \mathfrak{l}_2$ of two Lagrangian subspaces $\mathfrak{l}_1 \subset \mathfrak{a} \oplus \mathfrak{g}_{2\alpha}$ and $\mathfrak{l}_2 \subset \mathfrak{g}_\alpha$.

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Actually, $\mathfrak{s}' = \mathfrak{l}_1 \oplus \mathfrak{l}_2$ is isomorphic to the *canonical Lagrangian subalgebra* in $\mathfrak{s}$

$$\mathfrak{l}_\theta = \mathrm{span}_{\mathbb{R}}\{\cos\theta A + \sin\theta Z\} \oplus \mathrm{span}_{\mathbb{R}}\{X_1, \cdots, X_{n-1}\} \text{ for } \theta \in [0, \pi/2].$$

(where $\mathfrak{a} = \mathrm{span}_{\mathbb{R}}\{A\}$, $\mathfrak{g}_{2\alpha} = \mathrm{span}_{\mathbb{R}}\{Z\}$ and $X_i \in \mathfrak{g}_\alpha$ s.t. $[X_i, JX_i] = Z$.)

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Denote the connected subgroup of S whose Lie algebra $\mathfrak{l}_\theta$ by L_θ. Lemma implies any Lag orbit $S' \cdot o$ for $S' \subset S$ is isometric to some $L_\theta \cdot o$. By computing the mean curvature, we see $L_\theta \cdot o$ is not isometric to $L_{\theta'} \cdot o$ if $\theta \neq \theta'$. Namely,

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Theorem (Hashinaga-K. 17)

The set $\mathcal{C}(S)$ consisting of congruence classes of Lagrangian orbits obtained by connected subgroups of S is parametrized by $\theta \in [0, \pi/2]$, and $L_\theta \cdot o$ represents each congruence class.

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Furthermore, we determined the orbit equivalence class:

Theorem (Hashinaga-K. 17)

Let S' be a connected Lie subgroup of $S \simeq \mathbb{C}H^n$. If $S' \curvearrowright \mathbb{C}H^n$ admits a Lagrangian orbit, then the S'-action is orbit equivalent to either L_0 or $L_{\pi/2}$-action. Here,

- *L_0-action yields a 1-parameter family of Lag orbit including all congruence classes in $\mathcal{C}(S)$ except $[L_{\pi/2} \cdot o]$ ($\exists$unique totally geodesic orbit $L_0 \cdot o \simeq \mathbb{R}H^n$).*
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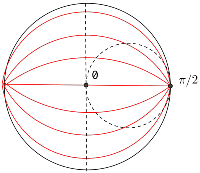

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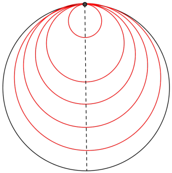

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Note: The orbit space of Lagrangian orbits can be described by the moment map $\mu : \mathbb{C}H^n \to (\mathfrak{s}')^*$:

- (roughly speaking)

$$\{\text{Lag } S'\text{-orbits}\} \ni \mu^{-1}(c) \longleftrightarrow c \in \mathfrak{z}((\mathfrak{s}')^*) = \{c \in (\mathfrak{s}')^* : \mathrm{Ad}^*(g)c = c \ \forall g \in S'\}$$

- For example, if $S' = L_0$, then $\mathfrak{z}((\mathfrak{s}')^*) = \mathbb{R}A^*$. Thus, taking $\gamma(t) \in S \simeq \mathbb{C}H^n$ s.t. $\mu(\gamma(t)) = tA^*$, $\gamma(t)$ intersects to every Lag L_0-orbits.