New examples of critical metrics for quadratic functionals

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Universidade de Santiago de Compostela

28-31 October 2019, Santiago de Compostela

Symmetry and Shape,
Celebrating the 60th birthday of Prof. J. Berndt

Notation

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- Einstein manifold $\Leftrightarrow \rho = \lambda g$, $\lambda \in \mathbb{R}$.

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Berger $\rightarrow$ expression of critical metrics for each functional
See M. Berger, Quelques formules de variation pour une structure riemannienne, *Ann. Sci. École Norm. Sup.* **3** (1970), no. 4, 285–294.

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Problem

Existence of non-Einstein critical metrics.

Aim

To describe all three-dimensional non-Einstein homogeneous critical metrics for all quadratic curvature functionals.

Homogeneous spaces

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Theorem

Let (M, g) be a complete and simply connected three-dimensional homogeneous manifold. Then it is either symmetric or isometric to a Lie group with a left-invariant metric.

See K. Sekigawa, On some 3-dimensional curvature homogeneous spaces, *Tensor (N.S.)* **31** (1977), 87–97.

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- Symmetric manifolds
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  - Non-Unimodular Lie groups
  - Unimodular Lie groups

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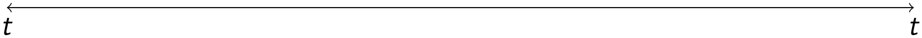
- Einstein $\leftrightarrow$ space form.
- $\tau = 0$. There are just two non-Einstein left-invariant metrics on S^3.

Homogeneous $\mathcal{F}_t$-critical metrics ($\mathcal{F}_t = \int_M \|\rho\|^2 + t\tau^2$)



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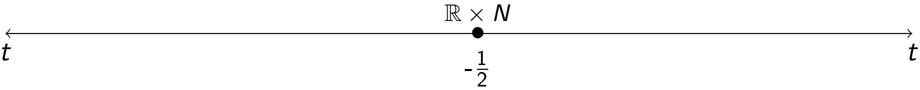

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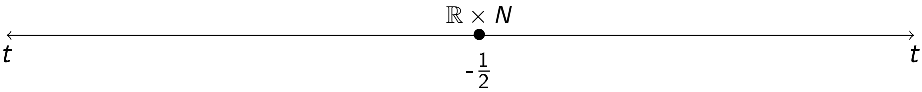

Non unimodular Lie groups

There exists an orthonormal basis $\{e_1, e_2, e_3\}$ of $\mathfrak{g}$ such that

$$[e_1, e_2] = \alpha e_2 + \beta e_3, \qquad [e_1, e_3] = \gamma e_2 + (2-\alpha) e_3, \qquad [e_2, e_3] = 0.$$

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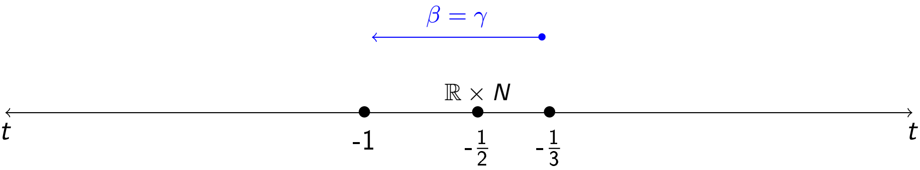

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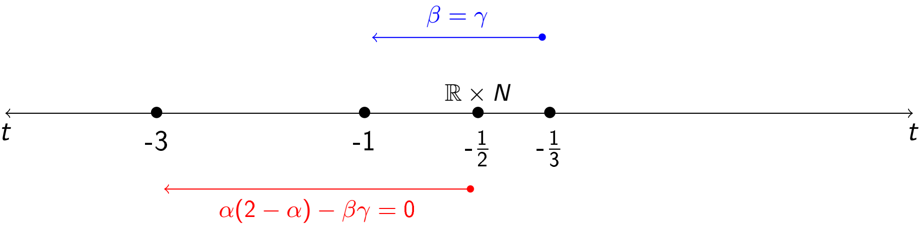

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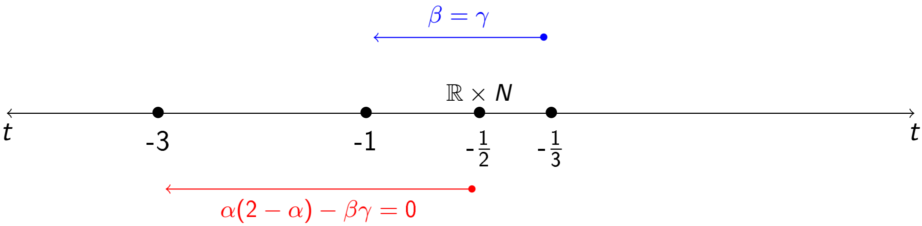

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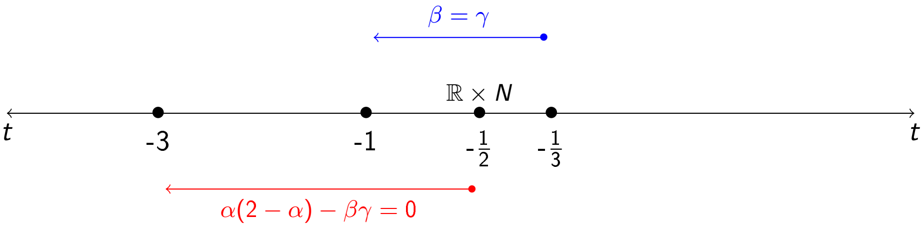

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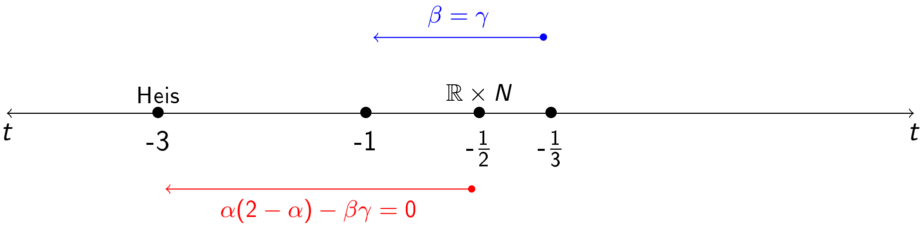

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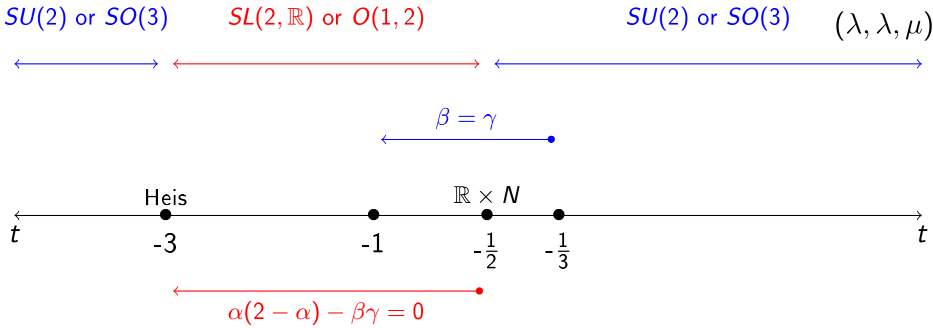

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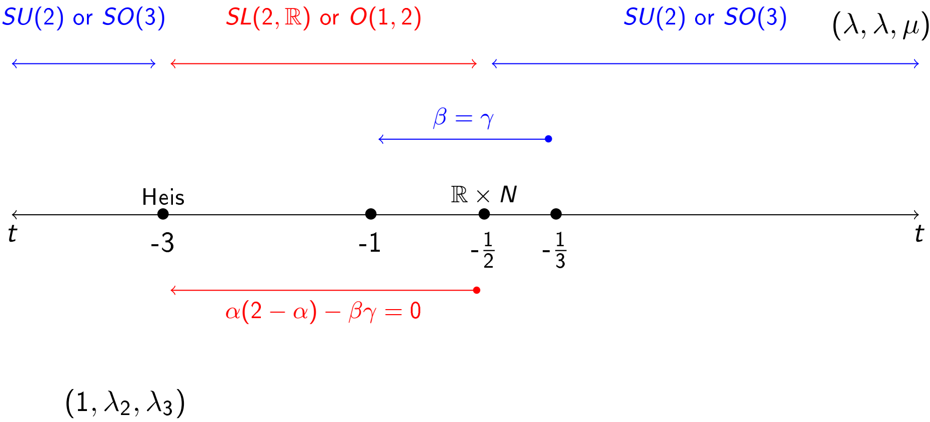

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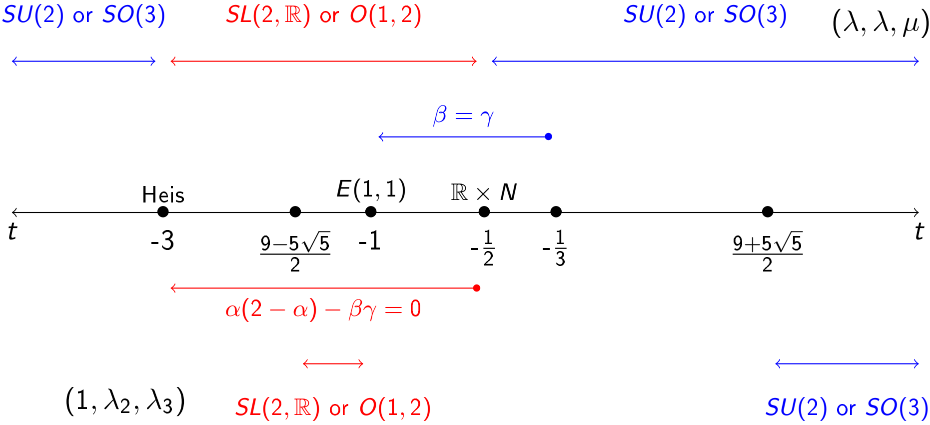

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- $t = 2$: **The spectral functional.** The spectral quadratic polynomial is determined by
$$S_2 = 2\|R\|^2 - 2\|\rho\|^2 + 5\tau^2 + 12\Delta\tau$$
$\mathcal{S}_2 = \int_M S_2 dvol_g$ is equivalent to $\mathcal{F}_2$.

Critical metrics:

$SO(3)$ with structure constants $(k, k, 2k)$ and Ricci operator $Ric = diag(0, 0, 2k)$.

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- $t = -2$: **The volumal functional.** The volumal quadratic polynomial is determined by
$$V_2 = 8\|\rho\|^2 - 3\|R\|^2 + 5\tau^2 - 18\Delta\tau$$
$\mathcal{V}_2 = \int_M V_2 \, dvol_g$ is equivalent to $\mathcal{F}_{-2}$.

Critical metrics:

- Non-unimodular Lie group satisfying $\alpha(2-\alpha) - \beta\gamma = 0$ and $\gamma = \beta \pm 2\sqrt{6}$; Ricci curvatures $(-16, -16, 12)$.
- $SL(2, \mathbb{R})$ or $O(1, 2)$ with structure constants $(k, k, -6k)$ and Ricci operator $Ric = k^2 diag(-24, -24, 18)$

Non-homogeneous examples: Cones

Definition

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- $F(\partial_r, \partial_r) = \frac{r^{-4}}{n+1} \left(\tau_N^2 - 2n(n-5)\tau_N + n^2(n-1)(n-9) \right) = 0$
- $F(e_i, e_j) = r^{-4}(\tau_N - n(n-1)) \left(-(\rho_N)_{ij} + \delta_{ij} \frac{\tau_N + (n-9)}{n+1} \right) = 0$

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Let (N, g_N) be a n-dimensional Riemannian manifold. The cone over N, $\mathbb{R}^+ \times_r N$, is $\mathcal{S}$-critical if and only if (N, g_N) is Einstein and its scalar curvature is

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- $\tau_N = n(n-9) \Rightarrow \tau = -\frac{8n}{r^2} \neq cte.$

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- $\tau_N = n(n-9) \Rightarrow \tau = -\frac{8n}{r^2} \neq cte.$
- $\dim N = 3$: $\mathbb{R} \times_r \mathbb{H}^3(-3)$ is $\mathcal{S}$-critical and $\mathcal{F}_t$-critical por all t.

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