Complete submanifolds of Euclidean space with codimension two

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Joint work with Cleidinaldo Silva – UFPI

Symmetry and Shape

Celebrating the 60th birthday of Prof. J. Berndt

Motivation

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Goal: To classify isometric immersions $f : M^n \to \mathbb{R}^{n+2}$ of a compact Riemannian manifold M^n of cohomogeneity one under the action of a closed Lie subgroup $G \subset \mathrm{Iso}(M)$ such that the principal orbits are umbilic hypersurfaces in M^n.

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Let $f : M^n \to \mathbb{R}^{n+1}$ be an isometric immersion of a compact homogeneous Riemannian manifold, i.e., $Iso(M)$ acts transitively on M. Then f embeds M^n as a round sphere.

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Let M^n be a compact Riemannian manifold under the action of a closed Lie subgroup $G \subset \mathrm{Iso}(M)$ with cohomogeneity one, and let $f : M^n \to \mathbb{R}^{n+1}$ be an isometric immersion.

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is an equivariant isometry with respect to the actions of G on the spaces $J \times_\rho G(p_0)$ and M_r.

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An isometric immersion $f : L^l \times_\rho M^m \to \mathbb{Q}_c^n$ is said to be a warped product of isometric immersions determined by a warped product representation $\phi : V^{n-k} \times_\sigma \mathbb{Q}_{\tilde{c}}^k \to \mathbb{Q}_c^n$, onto an open dense subset of $\mathbb{Q}_c^n$, if there exist isometric immersions $h_1 : L^l \to V^{n-k}$ and $h_2 : M^m \to \mathbb{Q}_{\tilde{c}}^k$ such that $\rho = \sigma \circ h_1$

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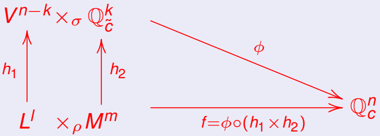

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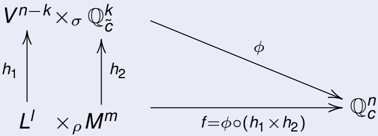

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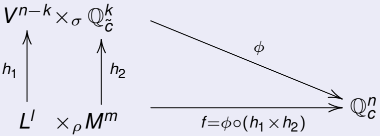

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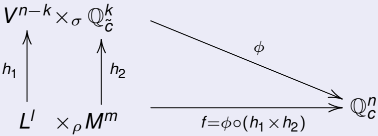

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$$\begin{array}{ccc} V^{n-k} \times_\sigma \mathbb{Q}^k_{\tilde{c}} & \overset{\phi}{\searrow} & \\ h_1 \uparrow \qquad \uparrow h_2 & & \mathbb{Q}^n_c \\ L^l \quad \times_\rho M^m & \xrightarrow[f=\phi\circ(h_1\times h_2)]{} & \end{array}$$

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If h_1 is a local isometry then, for $c = 0$, we have that $f(L^l \times_\rho M^m)$ is contained in the product of an Euclidean factor $\mathbb{R}^{n-k-1}$ with a cone in $\mathbb{R}^{k+1}$ over h_2.

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(i) There exist a compact homogeneous hypersurface $h : M^{n-1} \to \mathbb{S}^n_c$, a unit speed curve $\lambda : J = (a, b) \to \mathbb{R}^2_+$ and an isometry $\psi : J \times_\rho M^{n-1} \to M_r$ such that $f \circ \psi$ is the warped product of λ with h determined by a warped product representation $\Phi : \mathbb{R}^2_+ \times_\sigma \mathbb{S}^n_c \to \mathbb{R}^{n+2}$.

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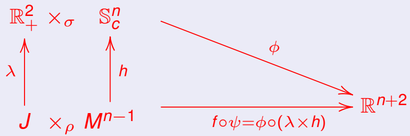

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(ii) There exist a compact surface $h : M^2 \to \mathbb{R}^4$ of intrinsic cohomogeneity one under the action of $\mathbb{S}^1$ and an isometry $\psi : M^2 \times_\rho \mathbb{S}_c^{n-2} \to M_r$ such that $f \circ \psi$ is the warped product of h with the identity map $i : \mathbb{S}_c^{n-2} \to \mathbb{S}_c^{n-2}$ determined by a warped product representation $\Phi : \mathbb{R}^4 \times_\sigma \mathbb{S}_c^{n-2} \to \mathbb{R}^{n+2}$.

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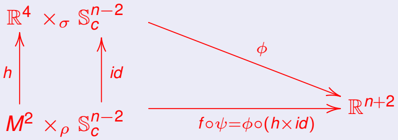

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Let $f : M^n \to \mathbb{R}^{n+2}$, with $n \geq 4$, be an isometric immersion of a compact Riemannian manifold of cohomogeneity one under the action of a closed Lie subgroup G of $\mathrm{Iso}(M)$. If the principal orbits under the action of G are umbilic hypersurfaces in M^n then one of the following possibilities holds:

(iii) There exist a unit speed curve $\lambda : J = (a, b) \to \mathbb{R}^2_+$ and an isometry $\psi : J \times_\rho \mathbb{S}_c^{n-1} \to M_r$ such that $f \circ \psi = F \circ G$, where G is the warped product of λ with the identity map $id : \mathbb{S}_c^{n-1} \to \mathbb{S}_c^{n-1}$ determined by a warped product representation $\phi : \mathbb{R}^2_+ \times_\sigma \mathbb{S}_c^{n-1} \to \mathbb{R}^{n+1}$, and $F : W \to \mathbb{R}^{n+2}$ is an isometric immersion of an open subset $W \subset \mathbb{R}^{n+1}$ that contains $G(J \times_\rho \mathbb{S}_c^{n-1})$.

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